Contract Enforcement and Young Firm Capital Structure: A Global Perspective*

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Abstract

We develop a framework to measure the severity of financial constraints for young firms across countries. Using ORBIS balance-sheet data for 23 economies, we show that short-term leverage rises while long-term leverage falls early in firms' life cycles, with this pattern persisting longer where contract enforcement is weaker. We build a model of optimal financing under limited enforcement with endogenous debt maturity and blueprint capacity that matches these patterns and enables structural measurement of financial constraints. The framework decomposes the funding gap into within-firm borrowing constraints that ease with repayment history and a scale distortion identifiable through cross-country comparisons.

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1 Introduction

How much of the difference in firm performance across countries is explained by financial frictions? The quantitative literature establishes that such frictions distort the allocation of capital and entrepreneurial talent, accounting for a substantial share of cross-country income and productivity gaps (Buera et al., 2011; Buera and Shin, 2013; Gopinath et al., 2017; Buera et al., 2023). Limited contract enforcement is a prominent source of these frictions, constraining financial intermediation and depressing the scale at which firms begin operations (Amaral and Quintin, 2010; Arellano et al., 2012). The quality of legal institutions is the deep determinant: stronger enforcement supports deeper financial markets and higher long-run income (La Porta et al., 1997, 1998; Levine, 2005). Yet measuring how severely constraints bind across institutional environments is difficult. Financial frictions are unobservable, and cross-country differences in firm size, growth, or employment reflect many forces beyond finance. To isolate their role, one needs an empirical object that is directly shaped by financing constraints and remains comparable across countries.

Capital structure is that object. Modigliani and Miller (1958) show that, absent frictions, financing decisions are irrelevant and leverage is indeterminate. Systematic patterns in leverage, debt maturity, or the composition of liabilities are therefore informative about the constraints firms face. Balance-sheet moments thus reveal the severity of financial frictions.

The same logic extends to cross-country analysis. Rajan and Zingales (1995) and Booth et al. (2001) show that the firm-level determinants of capital structure identified for U.S. firms carry over broadly to firms around the world. Yet financing outcomes vary systematically with country institutions beyond what firm characteristics alone predict. Firms in stronger-enforcement economies carry higher leverage and longer debt maturity, a pattern sharpest among young and small firms (Demirgüç-Kunt and Maksimovic, 1998; Beck et al., 2008; Qian and Strahan, 2007; Brown et al., 2012; Liu et al., 2024; van Solinge and Soederhuizen, 2023). The international corporate finance literature attributes this residual variation to enforcement institutions (Demirgüç-Kunt and Maksimovic, 1999; Fan et al., 2012) and we treat it as informative about the severity of financial constraints across countries. A countervailing force runs in the opposite direction: stronger creditor rights raise the threat of liquidation in distress, leading firms to choose lower leverage ex ante (Acharya et al., 2011). The positive enforcement-leverage relationship we document survives across multiple enforcement proxies, suggesting the supply channel dominates in our sample.

We focus on young private firms, among which financial frictions are especially consequential. Young firms account for a disproportionate share of job creation (Haltiwanger et al., 2013; Ayyagari et al., 2014), rely on external debt as their primary source of growth financing (Didier and Cusolito, 2024; Robb and Robinson, 2014), and are more likely than mature firms to face binding borrowing constraints (Hadlock and Pierce, 2010). Building on Albuquerque and Hopenhayn (2004) and Cooley et al. (2004), we model a young firm that signs an optimal contract with a competitive lender to finance a blueprint capacity chosen at entry, which we interpret as the scale of operations the firm commits to before it hires its first worker, and a sequence of working-capital needs. The firm can strategically default, so enforcement quality governs which repayment promises can be sustained. Two conditions deliver a unique optimal financial structure: a small fixed underwriting cost per contract, and first-period cash flows insufficient to cover both upfront and variable input costs. Consequently, long-term debt finances the blueprint capacity, and short-term debt finances working capital. This uniqueness eliminates the implementation multiplicity common in dynamic contracting models (Clementi and Hopenhayn, 2006; DeMarzo and Fishman, 2007; Verani, 2018) and delivers a direct mapping between model objects and the two debt categories observed in balance-sheet data.

The model implies distinct leverage dynamics over the early life cycle. Long-term leverage declines monotonically with age. The mechanism is not mechanical amortization: the firm optimally accelerates repayment to build continuation value, because a higher continuation value relaxes the enforcement constraint and expands future borrowing capacity. Short-term leverage rises while the firm remains constrained and falls once the constraint ceases to bind. Using ORBIS data for 22 European countries and Japan, we document a hump-shaped profile for short-term leverage and a monotone decline in long-term leverage. The age at which short-term leverage peaks is roughly two years later in low- relative to high-enforcement economies. Viewed through the lens of the model, this difference reveals a longer duration of the constrained phase in low-enforcement economies.

Measuring the full effect of contract enforcement on firm performance requires confronting a fundamental limitation of within-country evidence. Enforcement conditions at entry distort the scale at which firms begin operations. Firms anticipate future borrowing constraints when choosing their blueprint capacity, and a distorted scale choice in turn shapes the severity of those constraints going forward. Because firms in a given country operate under the same institutional environment, cross-country comparison is the natural moment to identify time-invariant choices like the scale distortion. Measuring financial constraint severity is therefore a cross-country exercise by necessity.

The paper makes two contributions. The first is theoretical. We develop, to our knowledge, the first model of optimal capital structure consistent with the cross-country patterns documented by the international corporate finance literature (Rajan and Zingales, 1995; Booth et al., 2001; Demirgüç-Kunt and Maksimovic, 1999; Fan et al., 2012). The model pins down maturity structure rather than leaving it indeterminate, which allows the joint dynamics of short- and long-term leverage over the early life cycle to be read as a direct signal of constraint severity. The framework also requires using firm age rather than size as the empirical margin for tracing constraint relaxation, a choice that remains valid in the presence of permanent productivity heterogeneity (Midrigan and Xu, 2014; Moreira, 2016; Sterk et al., 2021).

Our second contribution is empirical. We document a new fact: among young private firms, short-term leverage follows a hump-shaped profile over the early life cycle while long-term leverage declines monotonically. The prior literature studies total leverage over the full life cycle and consistently finds a declining age profile for private firms in the United States (Dinlersoz et al., 2018), France (Derrien et al., 2021), and Europe (Kochen, 2023). Focusing on the early life cycle and separating the two components reveals dynamics that aggregate measures obscure: the compositional shift between short- and long-term debt carries information about financial constraints that total leverage alone cannot. The empirical implementation draws on harmonized European firm-level balance-sheet data from ORBIS (Kalemli-Özcan et al., 2024), following a growing literature that uses this source to study financial frictions across countries (Gopinath et al., 2017; Arellano et al., 2012). From this foundation we construct a funding gap index that decomposes constraint severity into a within-firm component, which relaxes with repayment history, and a scale component, which reflects distortions present at entry. The index maps directly to real outcomes: we show how to translate the funding gap into an employment gap, and validate the implied magnitudes against Eurostat Business Demography data.

In the data, enforcement quality is positively associated with long-term leverage among young private firms: a one-percentage-point increase in the recovery rate is associated with a 0.41 percentage-point increase in long-term leverage, significant at the one percent level across 23 European countries. Our estimates imply a total low-to-high-enforcement working-capital funding gap of 0.115. About four-fifths reflects a persistent scale distortion; the remaining fifth reflects within-firm borrowing limits that relax with age. The decomposition is not confounded by unobserved heterogeneity, because the constraint multiplier is invariant to permanent productivity differences across firms. Under our preferred estimate of the working-capital financing share, the model implies a 7.4 percent employment gap between low- and high-enforcement economies, which accounts for roughly one quarter of the employment gap observed in Eurostat Business Demography data.

Finally, the paper resolves a tension between canonical endogenous-constraint models and the international evidence. In models where the initial financing need is a fixed unproductive setup cost, stronger enforcement mechanically lowers long-term leverage: a fixed face value of debt is divided by an expanding asset base. That prediction is counterfactual. In our framework, the initial financing decision is tied to productive firm scale, so better enforcement supports both greater borrowing capacity and higher long-term leverage. Endogenizing the scale choice is therefore not merely a modeling convenience. It is what allows capital structure to serve as an informative signal of financial constraint severity across countries.

The remainder of the paper is organized as follows. Section 2 documents leverage patterns across enforcement environments. Section 3 develops the baseline theory. Section 4 extends the model to a stochastic setting with firm heterogeneity. Section 5 tests the model's predictions using firm-level data. Section 6 quantifies financial constraint severity and decomposes the funding gap into within-firm and scale components. Section 7 concludes. Proofs, data construction, and robustness specifications are in the Online Appendix.

2 Leverage and Development across Countries

We begin by documenting a stylized fact that motivates our theoretical framework: young firms' leverage is particularly sensitive to a country's level of contract enforcement. We then show that this pattern poses a challenge for standard models of financial constraints, setting up the theoretical contribution that follows.

2.1 Data and Variable Construction

Our analysis uses firm-level data from the ORBIS database compiled by Bureau van Dijk, following the data cleaning methodology of Kalemli-Özcan et al. (2024). ORBIS provides harmonized balance sheet information for private firms across countries, which we supplement with institutional measures from the World Bank's Doing Business indicators.[1]

[1] We defer detailed discussion of sample construction and firm selection criteria to Appendix OA.1.

We define leverage ratios using a consistent denominator across all measures:

$$\text{Short-term Leverage} = \frac{\text{Short-term Debt}}{\text{Short-term Debt} + \text{Long-term Debt} + \text{Equity}}$$

$$\text{Long-term Leverage} = \frac{\text{Long-term Debt}}{\text{Short-term Debt} + \text{Long-term Debt} + \text{Equity}}$$

This formulation follows Rajan and Zingales (1995) and Welch (2011) in using total financing sources as the denominator, ensuring that leverage ratios remain invariant to changes in non-debt financing sources.[2]

2.2 Leverage Patterns and Institutional Quality

The international corporate finance literature documents that institutional quality shapes capital structure across countries: firms in economies with stronger enforcement use more long-term debt and carry longer maturities (Demirgüç-Kunt and Maksimovic, 1999; Fan et al., 2012; Demirgüç-Kunt and Maksimovic, 1998; Qian and Strahan, 2007). These findings are established using samples of publicly listed or large formal-sector firms pooled across all ages. Using ORBIS, Kochen (2023) shows that cross-country differences in leverage are more pronounced among younger firms, splitting the sample by income group. We complement this evidence by directly estimating the relationship between enforcement quality and leverage in the population of young private firms. Specifically, we estimate:

$$\text{Leverage}_{ict} = \alpha + \beta\,\text{Recovery Rate}_c + \mathbf{X}'_{it}\gamma + \alpha_{st} + \varepsilon_{ict}$$

where Recovery Rate_c is the percentage of debt recovered by secured creditors in insolvency proceedings (World Bank Doing Business, country-level average for 2003–2019), $\mathbf{X}_{it}$ includes firm-level controls (log labor compensation, ROA, tangibility), and α_{st} denotes sector×year fixed effects. The sample is restricted to young firms (age ≤ 11) drawn from 22 European countries and Japan over 1998–2021.

Table 1 confirms that the positive relationship between enforcement quality and leverage extends to young firms. A one-percentage-point increase in the recovery rate is associated with a 0.41 percentage-point increase in long-term leverage and a 0.43 percentage-point increase in total leverage, both significant at the 1% level. Since we use capital structure

[2] Equity is constructed as total assets minus current liabilities minus non-current liabilities, following Kochen (2023). Our qualitative results are unchanged when using shareholders' capital to measure equity.

TABLE 1: ENFORCEMENT QUALITY AND LEVERAGE: YOUNG FIRMS

	Long-term (1)	Total (2)
Recovery rate	0.413***	0.431***
	(0.037)	(0.031)
Firm controls	Yes	Yes
Sector×Year FE	Yes	Yes
Observations	16,114,998	16,114,998
R^2	0.134	0.134
Clusters	1,459	1,459

Notes: The dependent variable is leverage expressed in percentage points. Long-term leverage = LTD / (LTD + loans + equity); total leverage = (LTD + loans) / (LTD + loans + equity). Recovery rate is the World Bank Doing Business index (cents per dollar recovered in insolvency). The sample is restricted to young firms (age ≤ 11). Firm controls include log labor compensation, ROA, and tangibility. Standard errors clustered at the country×sector level. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

dynamics to identify financial constraints, establishing that young firms exhibit systematic cross-country differences in leverage composition is a prerequisite for our measurement strategy.

One concern is that recovery rates may partly reflect firms' endogenous demand for debt rather than the supply of credit. Acharya et al. (2011) show that stronger creditor rights can reduce corporate risk-taking, as the greater likelihood of liquidation in distress leads firms to choose lower leverage ex ante. To address this, Table 5 in Appendix A.1 replicates the analysis allowing for time series variation in four alternative World Bank contract enforcement indicators: the reorganization index, creditor participation index, recovery rate, and the resolving insolvency score. The positive association between enforcement and young-firm leverage is robust across all measures and specifications.

2.3 A Puzzle for Standard Models

The positive relationship between long-term leverage and enforcement quality documented in Table 1 poses a challenge for standard models of endogenous borrowing constraints.

In the canonical framework of Albuquerque and Hopenhayn (2004) and Cooley et al. (2004), firms finance a fixed, unproductive setup cost to begin operations. Long-term debt is pinned down by this cost and does not respond to enforcement quality. Stronger enforcement, however, relaxes borrowing constraints, raising short-term debt and the present value of future profits. Total assets grow while long-term debt stays fixed, so the long-term leverage ratio *falls* with enforcement quality. This is the opposite of what we observe in the data.

The counterfactual prediction arises because a fixed setup cost severs the link between enforcement and initial scale. Firms in low-enforcement economies start at the same scale as firms in high-enforcement economies; they simply face tighter constraints during the transition. In Section 3, we resolve this puzzle by replacing the unproductive entry fee with a productive investment in blueprint capacity. When blueprint capacity is a choice variable rather than a parameter, firms in low-enforcement economies optimally choose smaller capacity to economize on costlier external finance. Long-term debt then increases with enforcement along with firm scale, generating the positive relationship documented above. This modeling choice is motivated by evidence that financing constraints distort firms' entry scale: Cabral and Mata (2003) show that the evolution of the firm size distribution reflects constrained entrants starting below optimal scale rather than selection, and Arellano et al. (2012) show that cross-country differences in financial development generate observable differences in firm size dynamics.

3 Model

In this section, we develop a dynamic model of firm financing under limited enforcement. Firms borrow externally to fund production and face borrowing constraints that relax endogenously as they repay. The model delivers predictions about leverage dynamics that we test in subsequent sections. We begin with a deterministic environment to characterize the optimal financial contract and the duration of financial constraints. In Section 4, we introduce firm heterogeneity, which enables aggregation across firms and delivers the identifying moments for our empirical strategy.

3.1 Environment and Optimal Financial Structure

Firms operate a decreasing returns to scale technology with a quasi-fixed factor K (blueprint capacity) and a flexible factor n (labor). Blueprint capacity K is chosen at inception and remains fixed. Each period a has two subperiods. In the morning, firms begin production, pay for inputs, and save. At night, revenues are realized and firms choose whether to continue or strategically default; conditional on continuation, they pay dividends, save, and service debt. An exogenous exit shock kills the firm with probability $1 - \rho$ at period's end.[3] The

[3] We treat firm death as exogenous because ORBIS does not cleanly distinguish true exit from database attrition; Bajgar et al. (2020) show that measured exit in ORBIS confounds market exit with sample attrition.

within-period interest rate is $r_f > 0$, and the discount factor across periods is $\beta = (1+r_f)^{-2}$.

Productivity at age a is z_{ia}, initially treated as constant ($z_{ia} = z_i$) and publicly known. Section 4 extends this to $z_{ia} = \chi_i \tilde{z}_{ia}$, where χ_i is a permanent firm-specific component and $\tilde{z}_{ia}$ a transitory shock. The production function is $f(K, n) = z\,(K^\alpha n^{1-\alpha})^\eta$ with $\alpha, \eta \in (0, 1)$.

Three assumptions characterize the financial environment.

Assumption 1 (Property Rights) *An enforcement technology $\xi \in (0, 1)$ can be applied to contractual promises. Registering a contract requires a fixed cost $\kappa_C > 0$. The technology imposes a penalty $(1 - \xi)$ on strategic default but not on default induced by exit shocks.*

Assumption 2 (Transaction Costs) *Contracting costs satisfy $\kappa_C < S_0(\underline{\xi}, \underline{z})(1 - \rho)/\rho$, where $S_0(\underline{\xi}, \underline{z})$ is the short-term debt of the least productive firm in the lowest enforcement environment.*

Assumption 3 (Minimum Scale) *Parameters satisfy $(\beta\rho(1 + r_f) + \beta(1 - \rho))/((1 - \beta\rho)(1 + r_f)) > (1 - \eta)/(\eta\alpha)$.*

Proposition 1 (Financial Structure) *Under Assumptions 1–3, the optimal financial structure consists of long-term debt to fund blueprint capacity and a sequence of short-term loans to fund working capital needs.*

Long-term debt finances blueprint capacity K, while short-term debt finances working capital that fluctuates with production intensity. This maturity-matching result disciplines leverage measurement by providing clear empirical counterparts for both types of debt.

We denote long-term debt as $L = \{L_0; \{q_a\}_{a\geq 0}\}$ and short-term loans as $S = \{S_a; (1 + r_f)S_a\}_{a\geq 0}$, where q_a is the coupon payment at age a and S_a is short-term borrowing.

3.2 Firm's Problem Under Limited Enforcement

When enforcement is imperfect ($\xi < 1$), firms can engage in strategic default, keeping $(1 - \xi)$ of realized revenues. Financiers sign only enforceable contracts. The firm maximizes the expected present value of dividends:

$$V_0 = \max_{\{K, \{q_a, S_a, n_a\}_{a=0}^{\infty}\}} \sum_{a=0}^{\infty} (\beta\rho)^a \left(z \left[K^\alpha \left(\frac{S_a}{w} \right)^{1-\alpha} \right]^\eta - (1 + r_f)S_a - q_a \right)$$

We therefore focus identification on the model's capital-structure implications rather than on endogenous exit, which would be the natural alternative in the contracting framework of Albuquerque and Hopenhayn (2004).

subject to the participation constraint of financiers (PCF), the non-negativity constraint on coupons (NNC), and the limited enforcement constraint (LE):

$$p_k K(1+r_f) \leq \sum_{a=0}^{\infty} (\beta\rho)^a q_a \tag{PCF}$$

$$0 \leq q_a \leq z\left[K^\alpha \left(\frac{S_a}{w}\right)^{1-\alpha}\right]^\eta - (1+r_f)S_a \tag{NNC}$$

$$(1-\xi)z\left[K^\alpha \left(\frac{S_a}{w}\right)^{1-\alpha}\right]^\eta \leq V_a \quad \forall a \tag{LE}$$

where employment has been substituted using $n_a = S_a/w$ (Proposition 1) and w is the exogenous wage. The participation constraint ensures financiers recover the capital cost; the non-negativity constraint bounds coupons by available profits; and the enforcement constraint ensures the firm prefers repayment to strategic default at every age.

Proposition 2 (Optimal Financial Contract) *For any scale K, the optimal contract $\{S_a(K), L_a(K)\}_{a\geq 0}$ has the following structure. Long-term debt satisfies $L_a(K) = \sum_{j=a}^{\infty}(\beta\rho)^j q_j(K)$ with $L_0(K) = p_k(1+r_f)K$. Debt service is front-loaded: $q_a(K) = \pi_a(K)$ for $a < T(K)$, $q_a(K) = \gamma(K)\pi_a(K)$ at maturity $a = T(K)$ where $\gamma(K) \in (0,1]$, and $q_a(K) = 0$ for $a > T(K)$. Here $\pi_a(K) = z[K^\alpha(S_a(K)/w)^{1-\alpha}]^\eta - (1+r_f)S_a(K)$ denotes profits net of interest, and $T(K)$ is the maturity of long-term debt.*

Short-term borrowing increases during the early life cycle and reaches the unconstrained level at age $\hat{a}$:

$$S_a(K) = \begin{cases} \Theta_a(\gamma)\cdot S_u(K,z) & \text{if } a < \hat{a} \\ S_u(K,z) & \text{if } a \geq \hat{a} \end{cases}$$

where the constraint multiplier is

$$\Theta_a(\gamma) \equiv \left[\frac{(\beta\rho)^{T-a}[\beta\rho + (1-\beta\rho)(1-\gamma)]}{(1-\beta\rho)(1-\xi)}(1-\eta(1-\alpha))\right]^{\frac{1}{\eta(1-\alpha)}} \in (0,1)$$

and $S_u(K,z) = w\left[\frac{z\eta(1-\alpha)}{w(1+r_f)}\right]^{\frac{1}{1-\eta(1-\alpha)}} K^{\frac{\eta\alpha}{1-\eta(1-\alpha)}}$ is the unconstrained short-term debt level. Dividends satisfy $d_a = \pi_a - q_a$ and employment is $n_a = S_a/w$.

Front-loading of debt service minimizes the time during which the enforcement constraint binds. Dividends are correspondingly back-loaded: none are paid until debt is fully repaid.

Short-term debt rises until reaching the unconstrained level at age $\hat{a}$, while long-term debt declines as the firm retires it. We refer to the interval $[0, \hat{a}]$ as the Early Life Cycle (ELC). While the optimal contract is unique up to age $\hat{a}$, multiple implementations are consistent with optimality afterward; we adopt the front-loaded convention that maximizes balance sheet capacity.

3.3 Early Life Cycle Duration

The constraint multiplier $\Theta_a(\gamma)$ is below unity when

$$\beta\rho^{T-a} \cdot [\beta\rho + (1-\beta\rho)(1-\gamma)](1-\eta(1-\alpha)) < (1-\beta\rho)(1-\xi).$$

The ELC duration $\hat{a}$ is the age at which this inequality first fails:

$$\hat{a} = \left\lceil T - \frac{\ln\left(\frac{(1-\beta\rho)(1-\xi)}{(1-\eta(1-\alpha))[\beta\rho+(1-\beta\rho)(1-\gamma)]}\right)}{\ln(\beta\rho)} \right\rceil$$

where $\lceil \cdot \rceil$ denotes the ceiling function. ELC duration depends on enforcement quality ξ, the discount factor $\beta\rho$, production parameters η and α, and debt maturity T.

Enforcement affects ELC duration through two opposing forces. Taking the limit to handle the non-differentiable ceiling function and maturity $T(\xi)$:

$$\lim_{\epsilon\to 0}[\hat{a}(\xi+\epsilon) - \hat{a}(\xi)] = \lim_{\epsilon\to 0}[T(\xi+\epsilon) - T(\xi)] + \frac{1}{(1-\xi)\ln(\beta\rho)}.$$

The direct effect $1/[(1-\xi)\ln(\beta\rho)]$ is negative ($\ln(\beta\rho) < 0$): better enforcement relaxes constraints, allowing firms to exit the ELC sooner. The indirect effect through maturity is positive, as stronger enforcement enables longer maturities, which can extend the ELC.

Proposition 3 (Discrete Changes in Optimal Maturity) *The optimal maturity $T(\xi)$ is non-decreasing in enforcement quality ξ. For any continuous change in ξ, the change in optimal maturity satisfies $\Delta T(\xi) \equiv \lim_{\epsilon\to 0}[T(\xi+\epsilon) - T(\xi)] \in \{0, 1\}$.*

Since $\Delta T(\xi) \in \{0, 1\}$, the total effect is bounded:

$$\lim_{\epsilon\to 0}[\hat{a}(\xi+\epsilon) - \hat{a}(\xi)] \leq 1 + \frac{1}{(1-\xi)\ln(\beta\rho)}.$$

For typical parameter values ($\beta\rho \approx 0.5$, $\xi \geq 0.1$), this bound is negative: the direct effect dominates, so stronger enforcement reduces ELC duration.

3.4 Optimal Scale Under Limited Enforcement

We now turn to optimal scale determination. The non-differentiable structure requires solving an auxiliary problem over discrete pairs of scale K and maturity T.

Definition 1 (Feasible Discrete Scale and Maturity) *Let $\mathcal{F}$ be the set of pairs (T, K_T) satisfying the participation constraint (PCF) with equality, and let $V(T, K_T)$ denote the net present value of dividends for a firm with capital K_T, maturity T, and optimal capital structure as in Proposition 2.*

Proposition 4 (Optimal Scale and Maturity) *Let $\tilde{T} = \min\{T \in \mathbb{N} : V(T+1)/V(T) < 1\}$. The optimal scale solves $K^* = \arg\max_{K \in [K_{\tilde{T}}, K_{\tilde{T}+1}]} V(\tilde{T}+1, K)$, and the optimal maturity is $T^* = \tilde{T} + \mathbf{1}_{\gamma(K^*)>0}$. The repayment share $\gamma(K^*)$ and ELC duration $\hat{a}^*$ are determined endogenously from (T^*, K^*) consistent with Proposition 2.*

The trade-off involves balancing higher scale against earlier dividend payments. By choosing scale below the maximum feasible $K_{\tilde{T}+1}$, the firm can pay partial dividends at time $\tilde{T}+1$ rather than waiting until $\tilde{T}+2$.

Proposition 5 (Enforcement, Scale, and Maturity) *The feasible discrete scale K_T for any maturity T is strictly increasing in enforcement ξ. The optimal maturity $T^*(\xi)$ is weakly increasing in ξ. Economies with lower enforcement exhibit both lower optimal firm scale and shorter debt maturity.*

Scale affects the enforcement constraint proportionally on both sides: the scale effect (increased short-term borrowing needs) and the value effect (enhanced continuation value) are both proportional to K, causing them to cancel. The mechanism generates a sharp prediction: countries with stronger enforcement have firms with larger scale and longer maturities but shorter ELC durations.

3.5 Capital Structure and Leverage Dynamics

To match our empirical measures, define short-term and long-term leverage at age a:

$$\ell_a^S = \frac{S_a}{A_a} \quad \text{and} \quad \ell_a^L = \frac{L_a}{A_a}$$

where $A_a = V_a + S_a + L_a$ represents total assets, following Rajan and Zingales (1995) and Welch (2011).

Proposition 6 (Leverage Dynamics) *In an environment with enforcement quality* ξ*, leverage ratios evolve during the early life cycle* ($a < \hat{a}(\xi)$) *as follows:* $\Delta \ell_a^L \equiv \ell_{a+1}^L - \ell_a^L < 0$ *and* $\Delta \ell_a^S \equiv \ell_{a+1}^S - \ell_a^S > 0$.

Young firms start with high long-term leverage to fund blueprint capacity. As they age, they pay down long-term debt while simultaneously relaxing short-term borrowing constraints. These opposing movements continue until age $\hat{a}(\xi)$. The existing empirical literature finds that total leverage declines with age (Dinlersoz et al., 2018; Derrien et al., 2021; Kochen, 2023). Our model predicts, and our empirical analysis confirms, that short-term leverage *rises* during a firm's initial years while long-term leverage falls. This compositional shift is masked in aggregate leverage measures.

The deterministic model characterizes optimal contracts for a representative firm but has two limitations. Age and size are perfectly correlated, so the model cannot distinguish which dimension better identifies financial constraints (Cooley and Quadrini, 2001). Without a distribution of firms, we cannot construct the cross-sectional moments that map the model to data. The next section addresses both limitations by introducing stochastic productivity.

4 Firm Heterogeneity and the Role of Age

In this section, we introduce stochastic productivity. Randomness places a distribution over contract outcomes without altering the contract's functional form. Consequently, all balance sheet components are homogeneous of degree θ in permanent talent, so the constraint multiplier Θ_a is invariant to talent. These properties motivate a decomposition of the funding gap for firms into a within-country component, identified from age variation within a single economy, and a scale component that requires cross-country comparison.

4.1 Environment

Firms differ in permanent entrepreneurial talent $\chi_i \in \mathbb{R}_+$, drawn at entry from a distribution $F(\chi)$. Total productivity at age a is

$$z_{ia} = \chi_i \tilde{z}_{ia},$$

where $\tilde{z}_{ia}$ is transitory. The permanent component captures inherent differences in managerial ability; the transitory component captures demand fluctuations and other temporary factors.[4] Transitory shocks are i.i.d. across firms and time with $\mathbb{E}[\tilde{z}] = 1$ and finite moments $\mu_\theta \equiv \mathbb{E}[\tilde{z}^\theta], \mathbb{E}[\tilde{z}^{-\theta}] < \infty$, where $\theta = 1/(1 - \eta(1 - \alpha))$ is the productivity elasticity from Section 3. The multiplicative structure follows Sterk et al. (2021), who estimate that permanent heterogeneity accounts for the majority of cross-sectional variation in firm employment trajectories.

Transitory productivity $\tilde{z}_{ia}$ is realized at the start of the period corresponding to age a, before production and financial decisions are made. The contract conditions on the history $\tilde{z}^a \equiv (\tilde{z}_{i0}, \ldots, \tilde{z}_{ia})$: short-term borrowing $S_a(\tilde{z}^a)$, coupon payments $q_a(\tilde{z}^a)$, and the constraint multiplier $\Theta_a(\tilde{z}^a)$ depend on realized shocks.

4.2 Contract Structure and the Constraint Multiplier

The contracting problem extends Section 3 by requiring all constraints to hold state by state. Appendix OA.3 develops the recursive formulation and shows that the optimal contract retains the three-phase structure: a constrained phase with binding enforcement, a transition period, and an unconstrained phase. Each realized productivity history maps into a contract outcome:

$$\tilde{z}^\infty \longmapsto \big(\hat{a}(\tilde{z}^\infty),\, T(\tilde{z}^\infty),\, \gamma(\tilde{z}^\infty)\big). \tag{1}$$

Firms with favorable realizations retire debt faster and exit the ELC sooner. Stochastic productivity therefore induces a distribution over $(\hat{a}, T, \gamma)$ without changing the contract's functional form.

During the constrained phase, the enforcement constraint binds and the entrepreneur's continuation value equals the outside option, $(1 - \xi)\, y_a = V_a$. Current output y_a depends on the contemporaneous realization $\tilde{z}_{ia}$, while V_a depends on expected future profits and is proportional to expected unconstrained borrowing $\mathbb{E}[S_u(\tilde{z}, \chi, K)]$ (Appendix OA.3.4).

Write $S_a = \Theta_a \cdot S_u(\tilde{z}_{ia}, \chi, K)$, defining Θ_a as the fraction of desired borrowing at current productivity that the firm obtains. Substituting into the binding constraint and using

[4] The permanent component χ_i can also capture ex ante differences in entrepreneurial objectives, such as the distinction between growth-oriented and lifestyle entrepreneurs documented by Pugsley and Hurst (2011). Survey evidence in that paper shows that the median U.S. business owner does not intend to grow; such firms would correspond to low-χ_i draws in our framework.

homogeneity of S_u to cancel permanent talent and scale (Appendix OA.3.4):

$$\Theta_a(\tilde{z}^a) = \left[\frac{(1-\eta(1-\alpha))\, M_a(\tilde{z}^a)}{(1-\xi)(1-\beta\rho)} \cdot \frac{\mu_\theta}{\tilde{z}_{ia}^\theta}\right]^{\frac{1}{\eta(1-\alpha)}}, \tag{2}$$

where $M_a(\tilde{z}^a)$ is a maturity factor that summarizes repayment history and distance to contract maturity. No permanent firm characteristics survive: $\chi_i^\theta K^{\eta\alpha\theta}$ appears in both S_a (through V_a) and in S_u and cancels in the ratio.

The term $\mu_\theta/\tilde{z}_{ia}^\theta$ reflects the normalization by current-productivity borrowing capacity. The enforcement constraint pins feasible borrowing to expected continuation value, while S_u scales with the contemporaneous realization. When $\tilde{z}_{ia}$ is high, desired borrowing rises more than enforcement-limited capacity, so Θ_a falls.

The following decomposition sheds light on the determinants of the constraint multiplier in the stochastic model. Define $\Theta_a^{det} \equiv [(1-\eta(1-\alpha))\bar{M}_a/((1-\xi)(1-\beta\rho))]^{1/\eta(1-\alpha)}$, where $\bar{M}_a$ is the deterministic maturity factor. Then

$$\Theta_a(\tilde{z}^a) = \Theta_a^{det}(\xi, T, \gamma) \times \left[\frac{M_a(\tilde{z}^a)}{\bar{M}_a}\right]^{\frac{1}{\eta(1-\alpha)}} \times \left[\frac{\mu_\theta}{\tilde{z}_{ia}^\theta}\right]^{\frac{1}{\eta(1-\alpha)}}. \tag{3}$$

The first factor is the deterministic benchmark from Section 3. The second captures path dependence through repayment history. The third captures deviations between realized and expected productivity. When $\tilde{z}_{ia} = 1$ for all a, both stochastic adjustments equal one and $\Theta_a(\tilde{z}^a) = \Theta_a^{det}$.

Another important comparison between the two models concerns their predictions for firm exit. In the deterministic model, all firms exit the constrained regime at age $\hat{a}$. Under stochastic productivity, define the realized exit age as

$$\tilde{a}(\tilde{z}^\infty) := \min\{a \geq 0 : \Theta_a(\tilde{z}^a) \geq 1\} \wedge T.$$

We maintain the assumption that once a firm exits the constrained regime, it remains unconstrained.[5]

Finally, we can derive cross-sectional predictions. Let

$$\Phi(a) := \Pr(\tilde{a}(\tilde{z}^\infty) > a) = \Pr(\Theta_a(\tilde{z}^a) < 1)$$

[5] A sufficient condition is bounded shock support satisfying $\sup |\ln \tilde{z}| < \Delta g^{det}/(2\eta(1-\alpha))$, where Δg^{det} is the per-period increment of $\ln \Theta_a^{det}$.

denote the fraction of firms constrained at age a. In the deterministic model, $\Phi(a)$ drops to zero at $\hat{a}$. With shocks, $\Phi(a)$ declines smoothly as more firms exit the ELC. Average short-term leverage at age a reflects a mixture of constrained firms ($\Theta_a < 1$) and unconstrained firms ($\Theta_a = 1$). As $\Phi(a)$ approaches zero, average leverage flattens, producing a hump-shaped pattern.

Higher enforcement raises Θ_a^{det} at every age, shifting $\Phi(a)$ downward and concentrating exit ages at younger values. Leverage profiles therefore flatten earlier in high-enforcement countries.

4.3 Homogeneity, Identification, and Enforcement

The derivation above showed that talent and scale cancel from the constraint multiplier as an algebraic consequence of the binding enforcement condition. The following results state this formally.

Proposition 7 (Homogeneity) *Profits π_a, short-term debt S_a, long-term debt L_a, and firm value V_a are homogeneous of degree θ in the permanent productivity component χ_i.*

Corollary 1 (Talent Independence) *The constraint multiplier Θ_a and the contract outcomes $(\hat{a}, T, \gamma)$ depend on enforcement ξ and the transitory history $\tilde{z}^a$, but are independent of χ_i.*

Proofs appear in Appendix OA.3.4.1. The results imply that two entrepreneurs with different talent face identical constraint dynamics when they draw the same transitory path. Talent independence implies that leverage ratios $\ell_a^S \equiv S_a/A_a$ and $\ell_a^L \equiv L_a/A_a$ are invariant to permanent productivity, because both numerator and denominator scale with χ_i^θ. Leverage-age profiles are therefore informative about enforcement rather than the talent distribution. We exploit this property throughout the empirical analysis. The following proposition summarizes the main testable prediction.

Proposition 8 (Enforcement and Leverage) *For enforcement $\xi < \bar{\xi}$:*

(i) Short-term leverage is increasing in enforcement: $\partial \ell_a^S / \partial \xi > 0$.

(ii) Long-term leverage admits the decomposition

$$\ell_a^L = \frac{L_a}{L_a + V_a} \times (1 - \ell_a^S), \tag{4}$$

where the debt-to-capital ratio $L_a/(L_a + V_a)$ is strictly increasing in enforcement.

Better enforcement raises Θ_a and hence $S_a = \Theta_a S_u$. The decomposition (4) clarifies why long-term leverage also rises: enforcement increases debt capacity by reducing the value distortion induced by limited commitment. The second factor, $1 - \ell_a^S$, moves in the opposite direction, but for empirically relevant leverage ratios the debt-to-capital effect dominates.

4.4 The Funding Gap and Its Decomposition

To quantify the implications of leverage dynamics across countries, we characterize the total funding gap, which corresponds to the ratio of actual short-term borrowing to its first-best level. At age a, using $S_a = \Theta_a \, S_u(K^*, \chi, \tilde{z}_a)$ and the homogeneity of S_u in K:

$$\frac{S_a(K^*(\xi))}{S_u(K_u)} = \underbrace{\Theta_a}_{\text{within-firm}} \times \underbrace{\left(\frac{K^*}{K_u}\right)^{\eta\alpha\theta}}_{\text{scale distortion}}, \tag{5}$$

where talent, transitory productivity, and factor prices cancel out. The within-firm term Θ_a captures the severity of the binding enforcement constraint at a given scale and converges to one as constraints relax. The scale term captures the distortion in blueprint capacity induced by weak enforcement. Because blueprint capacity is fixed throughout the ELC, the scale component cannot be identified from within-country age variation alone.

Appendix OA.3.5.3 shows that optimal blueprint capacity satisfies

$$K^*(\xi, \chi) = K_u(\chi) \cdot \left(\frac{\Lambda(\xi)}{\Lambda^{FB}}\right)^{\frac{1}{(1-\eta)\theta}}, \tag{6}$$

where $\Lambda(\xi)$ summarizes the NPV of transfers from the firm to the lender, Λ^{FB} is its first-best counterpart under perfect enforcement, and all dependence on χ, w, p_k, and r_f is absorbed into $K_u(\chi)$. The ratio $\Lambda(\xi)/\Lambda^{FB} \in (0, 1]$ indexes how enforcement frictions reduce debt capacity. Substituting (6) into (5) yields a scale distortion of $(\Lambda(\xi)/\Lambda^{FB})^{\eta\alpha/(1-\eta)}$, which depends on enforcement alone.

Recovering the within-firm component Θ_a requires the counterfactual unconstrained level S_u, which is unobservable for constrained firms. To overcome this challenge, we use borrowing at the ELC exit age $\hat{a}$, where $\Theta_{\hat{a}} = 1$, as the benchmark. Specifically, we use the *ratio-of-means*

$$\Theta_a^{RM} \equiv \frac{\mathbb{E}[S_a]}{\mathbb{E}[S_{\hat{a}}]} \tag{7}$$

which averages short-term debt across firms at each age and then takes the ratio. By homogeneity (Proposition 7), $\chi_i^\theta K_i^{\eta\alpha\theta}$ cancels out. What remains is

$$\Theta_a^{RM} = \frac{\mathbb{E}[\Theta_a \cdot \tilde{z}_{ia}^\theta]}{\mu_\theta}, \tag{8}$$

an output-weighted average of the constraint multiplier that contains no prices, talent, or scale. Because the cross-section at age a includes both constrained and unconstrained firms, Θ_a^{RM} measures the aggregate shortfall at age a relative to the unconstrained benchmark. Finally, note that we can decompose Θ_a^{RM} into an intensive margin capturing the fraction of firms financially constrained $\Phi(a)$, and an intensive margin capturing the severity of financial constraints for the average firm at age a:

$$\Theta_a^{RM} = 1 - \Phi(a)\left(1 - \mathbb{E}[\Theta_a^{RM} | \Theta_a^{RM} < 1]\right) \tag{9}$$

The distortion scale is more difficult to identify. Specifically, the scale term in (5) cannot be recovered from within-country data because blueprint capacity is fixed throughout the ELC. To recover it, we need an auxiliary object for a country c,

$$R^c \equiv \frac{\mathbb{E}[L_0 \mid c]}{\mathbb{E}[S_{\hat{a}} \mid c]}, \tag{10}$$

which pairs initial long-term debt with unconstrained short-term debt at ELC exit. The cross-age pairing is designed so that the numerator contains no constraint multiplier, since L_0 is determined at entry before any constraint binds, while the denominator contains no enforcement distortion, since $S_{\hat{a}} = S_u(K^*, \chi, \tilde{z}_{\hat{a}})$ is evaluated at $\Theta_{\hat{a}} = 1$. Appendix OA.3.5.5 shows that wages, capital prices, the interest rate, and the talent distribution all share identical exponents in L_0 and $S_{\hat{a}}$ and therefore cancel out in the ratio, yielding $R^c \propto \Lambda(\xi_c)$. Cross-country ratios of R thus identify $\Lambda(\xi_c)/\Lambda(\xi_{c'})$ directly.

The scale distortion then follows by using this proportionality result in expression (6). The total effect of the scale distortion in (5) becomes:

$$\left(\frac{K^{*,c}}{K^{*,c'}}\right)^{\eta\alpha\theta} = \left(\frac{R^c}{R^{c'}}\right)^{\frac{\eta\alpha}{1-\eta}}. \tag{11}$$

4.5 The Steady-State Funding Gap Index

In order to implement the measure of distortions empirically, we aggregate expression (5) into a single steady-state index that integrates the depth of within-firm constraints, the duration of the constrained phase, and the scale distortion. Consider a low-enforcement economy c benchmarked against a high-enforcement economy c'. In steady state, the age distribution is geometric with survival probability ρ[6], and a fraction λ_c of entrants carry positive long-term debt. We restrict attention to the ELC ($a \leq \hat{a}^c$), where the model's predictions are sharpest and blueprint capacity is fixed.[7]

Definition 2 (Steady-State Funding Gap Index)

$$\mathcal{G}(c|c') \equiv \lambda_c \sum_{a=0}^{\hat{a}_c} (1-\rho)\rho^a \left[1 - \frac{\Theta_a^{RM,c}}{\Theta_a^{RM,c'}} \cdot \left(\frac{R^c}{R^{c'}}\right)^{\frac{\eta\alpha}{1-\eta}}\right]. \tag{12}$$

The index measures the fraction of efficient aggregate short-term credit missing during the ELC in steady state. It equals zero when the two economies share enforcement quality and increases as enforcement weakens. Note that the index can be decomposed as follows:

$$\mathcal{G}(c|c') = \mathcal{G}^{within}(c|c') + \mathcal{G}^{scale}(c|c'), \tag{13}$$

where

$$\mathcal{G}^{within}(c|c') \equiv \lambda_c \sum_{a=0}^{\hat{a}_c} (1-\rho)\rho^a \left[1 - \frac{\Theta_a^{RM,c}}{\Theta_a^{RM,c'}}\right], \tag{14}$$

$$\mathcal{G}^{scale}(c|c') \equiv \lambda_c \sum_{a=0}^{\hat{a}_c} (1-\rho)\rho^a \cdot \frac{\Theta_a^{RM,c}}{\Theta_a^{RM,c'}} \cdot \left[1 - \left(\frac{R^c}{R^{c'}}\right)^{\frac{\eta\alpha}{1-\eta}}\right]. \tag{15}$$

The within-firm component captures credit missing because the enforcement constraint binds at a given scale; this margin relaxes as firms accumulate repayment history. The scale component captures credit missing because firms entered at a distorted scale.

The index requires five inputs: the constraint profiles $\Theta_a^{RM,c}$ and $\Theta_a^{RM,c'}$, the scale ratio $\left(R^c/R^{c'}\right)^{\frac{\eta\alpha}{1-\eta}}$, the constrained-entrant share λ_c, ELC duration $\hat{a}_c$, and survival probability ρ. Technology enters only through $\eta\alpha$. Section 6 estimates these inputs and reports the decomposition.

[6] Consider an economy with a unit mass of firms. Each period, surviving firms continue with probability ρ and exiting firms are replaced by new entrants. The stationary age distribution is geometric: $\mu(a) = (1-\rho)\rho^a$. The survival weights $(1-\rho)\rho^a$ used throughout Section 6 follow directly.

[7] In an extended model with stochastic reinvestment arrivals, the scale distortion would decay at a rate determined by the arrival frequency and financing conditions for expansion. The ELC-restricted measure can be interpreted as a lower bound on total distortion.

Each component of $\mathcal{G}$ is a known function of the listed quantities. The ratio-of-means estimator identifies the output-weighted constraint multiplier, with permanent firm characteristics canceling by homogeneity (Proposition 7). The R moment identifies the relative debt capacity $\Lambda^*(\xi_c)/\Lambda^*(\xi_{c'})$, with the talent distribution and factor prices canceling out. The remaining objects are directly observable or can be estimated using data.

4.6 From Funding Gaps to Employment Gaps

In the model, S_a finances the entire flexible factor bill, which encompasses labor, intermediate inputs, and any other variable input funded through short-term borrowing. One dollar of missing short-term credit therefore maps one-for-one into a dollar of missing flexible-factor spending. In practice, only a fraction $\phi \in (0, 1]$ of the total wage bill is financed through short-term loans; the remainder is funded from internal cash flow and is not subject to the enforcement constraint. The per-firm employment shortfall at age a is therefore ϕ times the credit shortfall at that age, and the aggregate employment gap is

$$E_\phi(c|c') = \phi \cdot \mathcal{G}(c|c'). \tag{16}$$

The within-firm and scale decomposition carries over directly:

$$E_\phi(c|c') = \phi \cdot \mathcal{G}^{within}(c|c') + \phi \cdot \mathcal{G}^{scale}(c|c'). \tag{17}$$

Because ϕ scales both components equally, the within-firm and scale shares are invariant to ϕ. The implication for employment follows directly. The static labor demand condition and the enforcement constraint $S_a = \Theta_a S_u$ imply that the per-firm employment ratio satisfies (Online Appendix OA.4.1):

$$\frac{n_a^c}{n_a^{c'}} = 1 - \phi\left[1 - \frac{S_a^c}{S_a^{c'}}\right]. \tag{18}$$

This expression allows us to quantify the effects of financial constraints on firm employment.

4.7 Measurement concerns

The identification results above establish that factor prices, the talent distribution, and transitory productivity cancel out from both Θ_a^{RM} and R^c by homogeneity. Two residual confounders remain. First, both objects contain the productivity moment $\mu_\theta \equiv \mathbb{E}[\tilde{z}^\theta]$ in their denominators. The normalization $\mathbb{E}[\tilde{z}] = 1$ is imposed in all countries, with χ_i absorbing permanent productivity differences; under this normalization, μ_θ depends only on the dispersion of transitory shocks. Cross-country comparisons therefore require that transitory productivity dispersion is common across enforcement groups, so that μ_θ cancels in the ratios $\Theta_a^{RM,c}/\Theta_a^{RM,c'}$ and $R^c/R^{c'}$. Second, industry composition shifts the effective production parameters (η, α) and can generate sector-specific constraint dynamics. To address the latter, all cross-country comparisons use a common set of industry weights when computing S_a, so that differences in Θ_a^{RM} and R^c across enforcement groups reflect enforcement variation rather than differences in sectoral composition. Appendix A.7 contains details.

4.8 Relation to Existing Financial Constraint Indices

Most empirical indices classify firms by size. The Kaplan and Zingales (1997) index combines cash flow, leverage, Tobin's Q, dividends, and cash holdings. Whited and Wu (2006) propose an alternative based on cash flow, leverage, sales growth, and firm size. Hadlock and Pierce (2010) find that a parsimonious index based on size and age outperforms both.

Corollary 1 provides a structural rationale for this finding, at least for young firms. Because Θ_a is invariant to χ_i, size mixes talent and constraints: a small firm may be small because χ_i is low or because $\Theta_a < 1$. Age separates these channels. For Compustat firms, which have survived extensive selection, the variance of χ_i may be small enough that size-based indices remain informative. For the ORBIS sample, which includes the universe of registered firms, size-based classification would mechanically confound weak enforcement with low talent.

A separate tradition classifies listed firms by observed participation in credit markets. Faulkender and Petersen (2006) use the presence of a bond rating to sort firms, interpreting market participation as evidence that the friction is slack. Our model yields a related indicator. In the optimal contract, long-term debt finances blueprint capacity when the enforcement constraint binds. Its absence at entry signals that the constraint was slack at the firm's optimal scale. The model therefore predicts that the hump-shaped short-term leverage profile associated with constraint relaxation should appear only among firms that entered with long-term debt.

5 Life-Cycle Leverage Dynamics

The stochastic model generates testable predictions about how enforcement quality shapes young firms' capital structure over the early life cycle. Short-term leverage should rise with age as enforcement constraints relax, with the peak identifying the duration of the early life cycle (ELC). Countries with weaker contract enforcement should exhibit longer ELC durations, because weaker institutions slow the accumulation of repayment history required to exit the constrained regime. Long-term leverage should be higher in high-enforcement countries, reflecting larger optimal blueprint capacity. A separate prediction, that the ratio-of-means estimator identifies the output-weighted constraint multiplier, guides the measurement exercise in Section 6.

The third prediction connects to the cross-country pattern documented in Section 2: the positive relationship between long-term leverage and enforcement that standard models with fixed setup costs cannot produce. In our framework, stronger enforcement raises the optimal blueprint capacity K^*, which requires more long-term debt at entry.

This section tests the first two predictions using life-cycle leverage dynamics estimated from an age-period-cohort framework. Because the theoretical mechanism operates through the repayment of long-term debt and its gradual replacement by short-term borrowing, we restrict the sample to firms with positive long-term debt near entry, and document that this restriction isolates firms whose leverage dynamics conform to the model's predictions.

5.1 Empirical Strategy

We estimate leverage trajectories over the firm life cycle using an age-period-cohort (APC) framework that separates life-cycle effects from time-varying economic conditions and cohort-specific characteristics. Argente et al. (2024) apply this methodology to product life cycles and Kochen (2023) applies it to firm financing dynamics.

Following Argente et al. (2024), the baseline specification is

$$\text{Leverage}_{ict} = \sum_{a=1}^{A} \beta_a \cdot \text{AgeBin}_{it}^{a} + \sum_{k} \delta_k \cdot \text{Cohort}_k^N + \alpha_{cjt} + \varepsilon_{ict}$$

where Leverage_{ict} denotes the leverage ratio of firm i in country c at time t, and AgeBin_{it}^{a} is an indicator for firm i belonging to two-year age bin a at time t. The coefficients β_a capture age-bin effects relative to age bin 0–1. The terms Cohort_k^N are Deaton-normalized cohort dummies constructed separately within each country, consistent with the country-level variation in the period controls α_{cjt} (Deaton, 1997). The fixed effects α_{cjt} absorb all time-varying shocks at the country-sector-year level, so age effects are identified through within-cohort variation across calendar years. Standard errors are clustered at the country-sector level.[8]

The leverage measures follow Rajan and Zingales (1995) and Welch (2011) in using total financing sources as the denominator, as described in Section 2.1. This formulation ensures that leverage ratios are invariant to changes in non-debt financing and facilitates comparison with the theoretical predictions in Section 3.

As a robustness check, we complement the APC approach with firm fixed effects that identify age effects from within-firm variation. This specification eliminates concerns about selection or time-invariant unobserved heterogeneity. Results appear in Appendix A.3 and are qualitatively similar to the baseline estimates.

5.2 Identifying Financially Constrained Firms

Our analysis restricts the sample to firms that report positive long-term debt in either of their first two years of operation. We define an indicator $\text{LTD}_i = 1$ if firm i reports positive long-term debt in either of its first two observed years, and $\text{LTD}_i = 0$ otherwise. The restriction follows directly from the model: long-term debt finances blueprint capacity when enforcement constraints bind, so the enforcement mechanism that generates rising short-term leverage applies only to firms that use this instrument. Firms with $\text{LTD}_i = 0$ lack the observable financing arrangement through which the model's predictions operate, making it impossible to map their leverage dynamics to the enforcement channel. This is an observability argument, not a claim that $\text{LTD}_i = 0$ firms are unconstrained. We present evidence below that the two groups differ systematically on observable dimensions, and that $\text{LTD}_i = 1$ firms exhibit the growth and financing patterns characteristic of financially constrained firms in the literature.

Table 2 reports median total assets and return on assets by age bin and initial long-term debt status. Firms with $\text{LTD}_i = 1$ enter roughly 4 times larger in total assets (250,198 versus 61,925). This gap narrows over the life cycle but never closes: by ages 10–11, $\text{LTD}_i = 1$ firms remain more than twice as large. The two groups also differ in return on assets: $\text{LTD}_i = 0$ firms earn higher ROA at every age (1.63 versus 1.30 at entry). Under decreasing returns to scale, this pattern is a mechanical consequence of operating at smaller scale rather than an indicator of looser constraints. The persistence of the size gap despite higher average returns among $\text{LTD}_i = 0$ firms is consistent with different ex-ante scale choices across the

[8]The country-sector-year fixed effects also address cyclical variation in debt-equity composition across firm sizes, as documented by Begenau and Salomao (2018). Because these fixed effects absorb aggregate time-varying shocks within each country-sector cell, the age coefficients are not contaminated by differential cyclical responses of small and large firms.

two groups rather than a transitory constraint that firms outgrow.

Table 2: Median Firm Characteristics by Age Bin and Initial Long-Term Debt Status

Age Bin	Total Assets		ROA	
	LTD = 1	LTD = 0	LTD = 1	LTD = 0
0–1	250,198	61,925	1.30	1.63
2–3	366,653	110,922	1.24	1.42
4–5	456,229	154,504	1.16	1.33
6–7	535,000	194,000	1.09	1.29
8–9	610,760	241,269	1.03	1.25
10–11	676,270	298,777	0.96	1.21

Notes: ROA is defined as revenue over total assets. All entries are medians within each age bin and LTD status group.

To assess whether the two groups differ in their leverage trajectories, we estimate an augmented APC specification that interacts age-bin effects with the initial long-term debt indicator:

$$\text{Leverage}_{ict} = \sum_{a=1}^{A} \beta_a \cdot \text{AgeBin}_{it}^a + \sum_{a=1}^{A} \gamma_a \cdot \text{AgeBin}_{it}^a \times \text{LTD}_i + \sum_k \delta_k \cdot \text{Cohort}_k^N + \alpha_{cjt} + \varepsilon_{ict} \quad (19)$$

Figure 1 plots the results and Table 9 in Appendix A.4 presents the estimated coefficients. Panel (a) plots predicted short-term leverage separately for $\text{LTD}_i = 1$ and $\text{LTD}_i = 0$ firms. $\text{LTD}_i = 1$ firms exhibit the hump-shaped profile predicted by the model: short-term leverage rises over the first several age bins as enforcement constraints relax, then levels off. $\text{LTD}_i = 0$ firms display a flat profile throughout. Panel (b) plots the difference, which widens significantly over the early life cycle. The flat profile among $\text{LTD}_i = 0$ firms indicates that these firms do not exhibit the enforcement-driven dynamics the model describes. This does not rule out that some $\text{LTD}_i = 0$ firms face other forms of credit constraints, but it implies that including them would introduce variation unrelated to the enforcement mechanism under study.

Taken together, the evidence indicates that $\text{LTD}_i = 1$ firms exhibit the hallmarks of financially constrained firms in an enforcement-based framework: they take on long-term debt to finance blueprint capacity, display rising short-term leverage as constraints relax, and grow in a manner consistent with the catch-up dynamics documented across countries at different levels of financial development (Arellano et al., 2012). $\text{LTD}_i = 0$ firms display

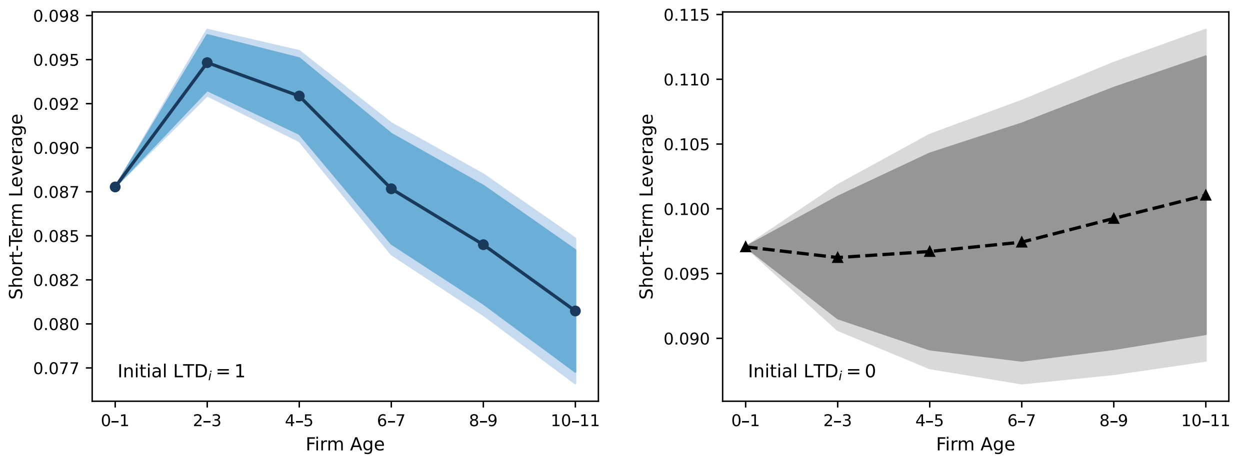

Figure 1: Short-term leverage by initial long-term debt status

Note: Panel (a) plots predicted short-term leverage from specification (19) separately for firms with $\text{LTD}_i = 1$ (solid, navy) and $\text{LTD}_i = 0$ (dashed, black). Panel (b) plots the difference. Age bin 0–1 is the reference category; predicted levels are anchored at the conditional group mean at ages 0–1 from the joint regression. Shaded regions denote 90 and 95 percent confidence intervals based on within-group standard errors from separate subsample regressions. All specifications include country×sector×year fixed effects and Deaton-normalized cohort controls. Standard errors clustered at the country-sector level.

none of these patterns. Their behavior is consistent with lifestyle entrepreneurship (Pugsley and Hurst, 2011), though we cannot rule out that some face constraints operating through channels other than enforcement. Including them would introduce heterogeneity unrelated to the mechanism that generates our theoretical predictions. All subsequent results restrict the sample to $\text{LTD}_i = 1$ firms.

5.3 Life-Cycle Patterns in Capital Structure

Figure 2 plots APC estimates of leverage profiles for the $\text{LTD}_i = 1$ sample and Table 10 in Appendix A.4 presents the coefficient estimates.

The left panel shows that short-term leverage exhibits a hump-shaped pattern, rising during the first years and peaking around age bin 4–5. This pattern is consistent with Proposition 6: young firms face binding enforcement constraints that limit working capital financing, and these constraints relax as firms accumulate repayment histories. The peak identifies the end of the early life cycle in the pooled sample.

The right panel shows that long-term leverage declines monotonically with age, reflecting the front-loaded repayment structure in Proposition 2. Total leverage also declines with age, but more gradually, as rising short-term leverage partially offsets falling long-term leverage during the early years.

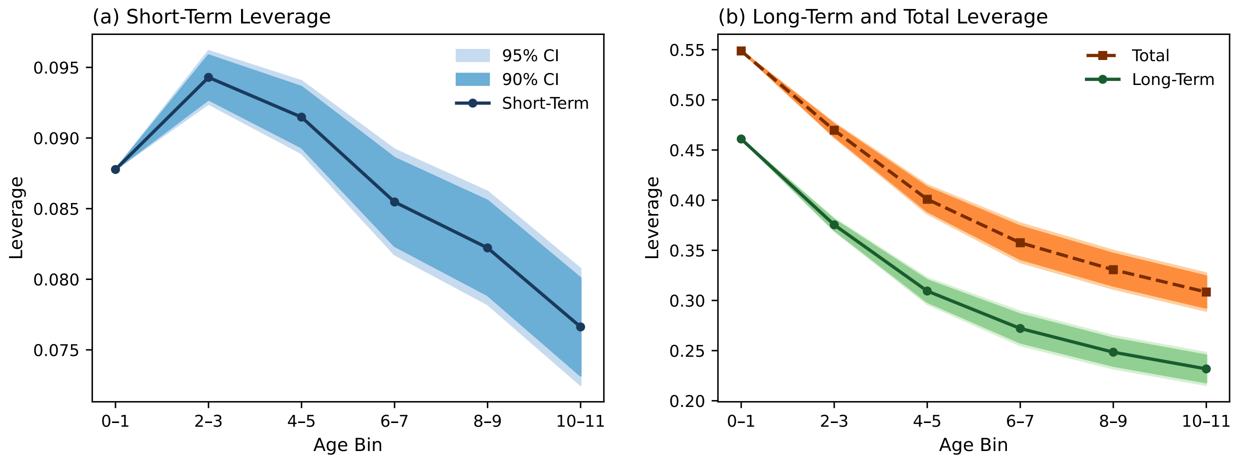

Figure 2: Leverage Over the Early Life Cycle

Notes: Coefficients from age-period-cohort regressions with country-sector-year fixed effects and Deaton normalization. Age bin 0–1 serves as the reference category. Shaded regions represent 90% and 95% confidence intervals. Standard errors clustered at the country-sector level. Sample includes firms with $\text{LTD}_i = 1$, observed for at least three years.

These opposing dynamics confirm the value of examining debt composition rather than total leverage alone. The aggregate decline in total leverage documented in the existing literature (Dinlersoz et al., 2018; Derrien et al., 2021; Kochen, 2023) masks a compositional shift that is central to identifying financial constraints.

5.4 Cross-Country Variation in Early Life Cycle Duration

The pooled estimates in Figure 2 average across countries with different institutional environments. The model predicts that weaker contract enforcement prolongs the early life cycle. We test this prediction by estimating separate APC models for country groups classified by recovery rates from the World Bank's Doing Business indicators (World Bank, 2020), which measure the percentage of debt recovered by secured creditors in insolvency proceedings. The list of countries and recovery rates can be found in Table 6 in Appendix A.2.

Figure 3 displays short-term leverage profiles for three enforcement groups: low (recovery rates below 45%), medium (45%–75%), and high (above 75%). Table 11 in Appendix A.4 reports the results from the estimation as well as the empirical test of the difference in peaks across groups. In low-enforcement countries, short-term leverage continues to rise significantly through age bin 4–5, indicating that firms remain constrained well into their first decade. In medium- and high-enforcement countries, the last statistically significant increase occurs by age bin 2–3. Firms in stronger institutional environments exit the early

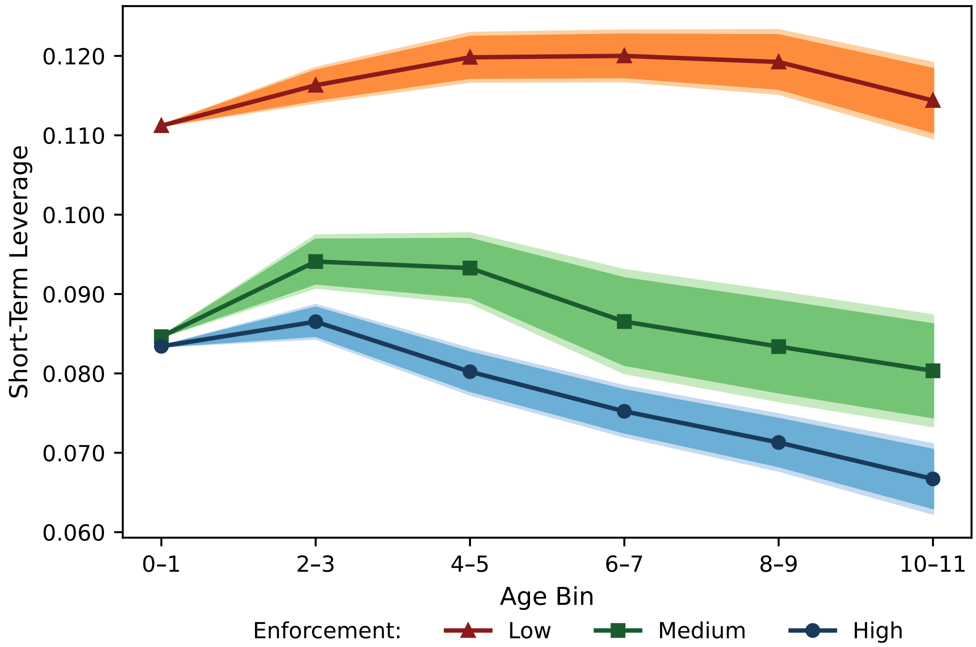

Figure 3: Short-Term Leverage Life Cycle by Recovery Rate Group

Notes: Predicted short-term leverage profiles by firm age for country groups classified by recovery rates: low (below 45%), medium (45%–75%), and high (above 75%). Age bins are two-year intervals. Shaded regions denote 90% confidence bands.

life cycle approximately two to four years sooner.

The profiles also differ in slope. High-enforcement countries exhibit flatter trajectories: firms start closer to their target leverage and require smaller adjustments over time. Low-enforcement countries exhibit steeper profiles, with firms gradually building leverage capacity as constraints relax. This pattern is consistent with Proposition 8: stronger enforcement enables higher leverage from inception, reducing the gap between initial and target capital structure.

Contract enforcement thus shapes both the duration and the slope of the leverage trajectory during the early life cycle. These patterns define the unconstrained benchmarks used in the next section to estimate constraint tightness and quantify the employment cost of weak enforcement.

6 Measuring Financial Constraints

A key finding from the empirical analysis summarized in Figure 3 is that firms in high- and medium-enforcement countries reach unconstrained status in age bin 2-3, while firms in low-enforcement economies do so in age bin 4-5. We use these estimates together with ORBIS data and the estimators developed in Section 4 to measure the severity of firm-level financial constraints across countries.

6.1 Constraint Tightness and Scale Distortions

Table 3 reports constraint tightness and scale distortions for firms with positive initial long-term debt ($\text{LTD}_i = 1$), the population modeled in Sections 3–4.

TABLE 3: CONSTRAINT TIGHTNESS AND SCALE DISTORTIONS DURING ELC

	High Enforcement (RR > 75%)	Medium Enforcement (45% < RR ≤ 75%)	Low Enforcement (RR ≤ 45%)
Panel A: Constraint tightness, Θ_a^{RM}			
Age $0-1$	0.753	0.750	0.690
$\Phi(0-1)$	73.0	73.8	73.9
Age $2-3$	1.000	1.000	0.846
$\Phi(2-3)$	39.4	39.8	64.5
Age $4-5$	—	—	1.000
$\Phi(4-5)$			39.8
Panel B: Scale distortion relative to High Enforcement			
R^c/R^{HE}	1.000	0.697	0.677
$(K^{*,c}/K^{*,HE})^{\eta\alpha\theta}$	1.000	0.649	0.626

Notes: Θ_a^{RM} is the ratio of mean short-term debt at age a to mean short-term debt at age $\hat{a}$. $\hat{a}$ is age bin 2-3 for HE and ME, and 4-5 for LE. For each country group, R^c is ratio of mean long-term debt at age bin 0-1 to mean short-term debt at age $\hat{a}$. $\Phi(a)$ is percentage of firms with short-term debt below unconstrained level at age bin a. c denotes country group and HE denotes benchmark high-enforcement group. Parameter choices are $\eta = 0.80$, $\alpha = 0.30$. All means are computed first at the country level with common industry weights, and then aggregated across countries in the group using GDP-weights. Sample: ORBIS, 22 European countries and Japan, firm with positive long-term debt at age bin 0-1.

Panel A separates constraint severity into depth and prevalence. At ages 0–1, low-enforcement firms access 0.690 of unconstrained borrowing capacity, compared to 0.753 in high enforcement. The share of constrained firms is only marginally higher in low enforcement economies (73.9% versus 73%), so most of the difference between low and high enforcement operates through the depth of constraints rather than their incidence.

Positive initial long-term debt is necessary but not sufficient for the financing constraint to bind. The condition $L_0 > 0$ selects firms that use long-term debt contracts at entry, the population for which enforcement affects the contract objects measured here. Among these firms, the leverage dynamics identify the subset that remains constrained: firms with $\Theta_a^{MR} < 1$. At ages 0–1, about 30% of firms with positive long-term debt at birth already exhibit $\Theta \geq 1$ in each enforcement group, indicating that they entered with long-term debt but are not constrained at their chosen scale.

Constraint relaxation differs sharply across countries. High- and medium-enforcement economies reach unconstrained status by age bin 2–3 ($\Theta_a^{RM} = 1.000$). Low-enforcement economies remain constrained at ages 2–3 ($\Theta_a^{RM} = 0.846$), with 64.5% of firms still below full capacity, and reach the unconstrained frontier only by age bin 4–5. Weak enforcement is therefore associated with a longer constrained phase.

Panel B reports the scale distortion measured by the ratio of long-term debt at birth to short-term debt at $\hat{a}$, which is summarized in expressions (10) and (11). Under standard parameter choices ($\eta = 0.8$ and $\alpha = 0.3$), the implied scale factors are 0.649 for medium enforcement and 0.626 for low enforcement.[9] Unlike the within-firm component, which relaxes as firms build repayment histories, the scale distortion is permanent: it reflects the blueprint-capacity choice and enters the working-capital ratio multiplicatively at every age of the ELC.

6.2 Funding and Employment Gap

We apply the funding gap index (Definition 2) to the estimates in Table 3, conditioning on firms with positive initial long-term debt ($L_0 > 0$). The per-period survival probability $\rho = 0.90$ corresponds to a biennial exit rate of ten percent, consistent with young-firm exit rates in the ORBIS sample (Bajgar et al., 2020). Panel A of Table 4 reports the decomposition.

The funding gap between low- and high-enforcement country groups amounts to 0.115. The scale component (0.093) accounts for 80.7 percent of the difference, and reflects the permanent shortfall from entering at sub-optimal blueprint capacity. The within-firm component (0.022) accounts for the remaining 19.3 percent and captures the differential borrowing constraint that relaxes with repayment history. The medium- versus high-enforcement group comparison isolates the scale channel: the two groups exhibit nearly identical constraint wedges, so the entire gap of 0.095 reflects scale distortions at entry. This observation

[9] We set $\alpha = 0.3$ so that labor's share of variable-input costs is $1 - \alpha = 0.70$. $\eta = 0.80$ is interpreted as revenue returns to scale. Under monopolistic competition with physical returns $\eta_{\text{phys}} = 0.95$, consistent with production function estimates for European manufacturing (Doraszelski and Jaumandreu, 2013), the formula $\eta = \eta_{\text{phys}} \times (\sigma - 1)/\sigma$ implies $\sigma \approx 6.3$, a demand elasticity in the standard range of the heterogeneous-firm literature.

Table 4: Funding Gap and Employment Gap

	Medium Enforcement $45\% < \text{RR} \leq 75\%$	Low Enforcement $\text{RR} \leq 45\%$
Panel A: Funding gap $\mathcal{G}(c \mid HE)/\lambda_c$		
Within-firm component, $\mathcal{G}^{within}$	0.000	0.022
Scale component, $\mathcal{G}^{scale}$	0.095	0.093
Total	0.095	0.115
within-firm (%)	0.4	19.3
scale (%)	99.6	80.7
Panel B: Employment gap		
Model-implied: $\hat{\varphi} \cdot \mathcal{G}/\lambda_c$	0.061	0.074
Eurostat observed: $\hat{\mathcal{E}}$	0.154	0.275
enforcement channel (%)	39.5	26.9
residual (%)	60.5	73.1

Notes: Panel A: $\mathcal{G}^{within}$ and $\mathcal{G}^{scale}$ are defined in expressions (14)–(15); HE denotes the benchmark group (RR > 75%). Country-level means use common industry weights; group aggregates use GDP weights. λ_c denotes the share of entrants with $L_0 > 0$. Panel B: model-implied gap is $\hat{\phi} \cdot \mathcal{G}/\lambda_c$ with $\hat{\phi} = 0.64$ (grand mean; Table 14). Eurostat gap is $\hat{\mathcal{E}} = \sum_b \tilde{\omega}_b [1 - n_b^c / n_b^{HE}]$, where $n_b^c = \texttt{EMP\_SRVL}(b)/\texttt{ENT\_BIRTH}$ is GDP-weighted across countries in group c and $\tilde{\omega}_b = \sum_{a \in b}(1-\rho)\rho^a$ normalized over ages 0–5. HE excludes Ireland; LE excludes Greece; ME excludes France. The enforcement channel is $\hat{\varphi} \cdot \mathcal{G}/\lambda_c$; the residual is $\hat{\mathcal{E}} - \hat{\varphi} \cdot \mathcal{G}/\lambda_c$. Calibration: $\eta = 0.80$, $\alpha = 0.30$, $\rho = 0.90$. Source: ORBIS (Panel A); Eurostat Business Demography (Criscuolo et al., 2014), cohorts 2004–2014 (Panel B).

strengthens the argument that a measurement strategy relying solely on within-country short-term debt dynamics would miss the dominant share of the cross-country working-capital difference.[10]

We examine employment next using expression (18). In Table 14 in Appendix A.6, we estimate $\hat{\phi} = 0.64$ from short-term debt and wage bill data for a subsample of the unconstrained firms in our sample, i.e. those aged 4 to 11 years. Because ϕ enters multiplicatively, the decomposition shares carry over from Panel A unchanged. At $\hat{\phi} = 0.64$, the model implies a 7.4 percent employment gap between low- and high-enforcement economies and 6.1 percent for medium- relative to high-enforcement countries.

To validate these magnitudes against an independent source, we construct an employment gap from Eurostat Business Demography cohort statistics (Criscuolo et al., 2014). Appendix A.8 describes the database and the detailed computations, which follow closely the methodology outlined in this section.

The observed gap between high- and low-enforcement groups in the Eurostat data is denoted by $\hat{\mathcal{E}}$ in Table 4 and amounts to 27.5 percent. The model-implied intensive-margin gap is $\hat{\phi} \cdot \mathcal{G}^{LE}/\lambda$, which equals 7.4 percent. Thus, the enforcement channel accounts for 26.9 percent of $\hat{\mathcal{E}}$, with the remainder attributable to other frictions that the model abstracts from. For medium-enforcement economies, $\hat{\mathcal{E}} = 15.4$ percent, so the enforcement channel accounts for 39.5 percent of it. In sum, financial frictions due to enforcement differences accounts for roughly one quarter of observed employment gaps for young firms across countries.

7 Conclusion

How much of the difference in firm performance across countries is explained by financial frictions? This paper develops a framework to answer that question using the capital structure of young private firms across countries with different enforcement institutions.

We document that short-term leverage rises while long-term leverage falls over the early life cycle, and that this co-movement persists longer where enforcement is weaker. A model of optimal financing under limited enforcement rationalizes these dynamics and reveals two channels through which constraints reduce firm size: within-firm borrowing limits that relax

[10] Cabral and Mata (2003) show that the evolution of the Portuguese firm-size distribution is consistent with financially constrained entrants growing toward their optimal scale. Their framework identifies the *dynamics* of constraint relaxation but cannot recover the *level* of the scale distortion relative to an unconstrained benchmark, since all firms share the same institutional environment. Cross-country comparison is required to identify that level.

with repayment history, and a permanent scale distortion. The scale channel cannot be detected from domestic data alone, making cross-country comparison necessary to measure the full severity of constraints.

The framework delivers a quantitative measure of constraint severity, expressed in units of working capital per firm. We estimate that scale distortions account for about four-fifths of the funding gap between low- and high-enforcement European economies. Under our preferred parametrization, the enforcement channel accounts for roughly one quarter of the employment gap between low- and high-enforcement economies observed in Eurostat Business Demography data.

Several limitations suggest directions for future work. The model features a single investment at firm creation; repeated investment would enrich the dynamics beyond the early life cycle. We focus on debt; equity and informal credit may matter where neither channel is well developed. The translation from working capital to employment depends on the working-capital financing share, which varies across enforcement regimes in ways that are difficult to separate from measurement differences in the data. Finally, our estimates are for the average firm across industries, and extending the analysis to the industry level is a natural next step.

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A Appendix

This appendix reports supplementary tables, figures, and estimation details that support Sections 5 and 6.

A.1 Robustness to Measures of Enforcement

In this section, we re-examine the relationship between firm leverage and enforcement using a variety of measures of contract enforcement from the World Bank Doing Business Database. We use four different indicators during the 2004-2020 period. Contrary to the main text, in this analysis, we utilize the entire time-series and cross-sectional variation in the enforcement measures. The results are summarized in Table 5 and are consistent with findings in the main text that firm long and total leverage are higher in countries with stronger enforcement.

Table 5: Enforcement Quality and Leverage: Young Firms

	Reorganization		Creditor participation		Recovery rate		Resolving insolvency	
	Long	Total	Long	Total	Long	Total	Long	Total
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Enforcement index	3.946***	8.103***	3.494***	2.677***	0.077***	0.096***	0.072***	0.089***
	(0.530)	(0.516)	(0.401)	(0.480)	(0.017)	(0.020)	(0.016)	(0.018)
Firm controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	14,189,668	14,189,668	14,189,668	14,189,668	13,895,428	13,895,428	13,895,428	13,895,428
R^2	0.242	0.194	0.242	0.192	0.247	0.195	0.247	0.195
Clusters	1,392	1,392	1,392	1,392	1,420	1,420	1,420	1,420

Notes: The dependent variable is leverage expressed in percentage points. Long-term leverage = LTD / (LTD + loans + equity); total leverage = (LTD + loans) / (LTD + loans + equity). Each column pair uses a different World Bank Doing Business enforcement index: reorganization index (1)–(2), creditor participation index (3)–(4), recovery rate (5)–(6), and resolving insolvency score (7)–(8). The sample is restricted to young firms (age ≤ 11). Firm controls include log labor compensation, ROA, and tangibility. Standard errors clustered at the country×sector level. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

A.2 Recovery Rates and Country Groupings

A.3 Within-Firm Evidence

This section supports the life-cycle leverage patterns reported in Figure 2 of Section 5 by showing that the age profiles persist with firm fixed effects. Figure 4 reports results from the

Table 6: Recovery Rates by Enforcement Group

Low Enforcement (RR ≤ 46)			*Medium Enforcement (46 < RR ≤ 76)*			*High Enforcement (RR > 76)*		
Country	ISO	RR	Country	ISO	RR	Country	ISO	RR
Ukraine	UA	9.5	Poland	PL	47.9	Sweden	SE	76.7
Romania	RO	14.3	France	FR	54.9	Germany	DE	81.5
Croatia	HR	30.8	Italy	IT	61.7	Iceland	IS	81.9
Bosnia-Herz.	BA	35.2	Portugal	PT	69.4	UK	GB	86.6
Estonia	EE	38.7	Spain	ES	73.3	Ireland	IE	87.1
Czech Rep.	CZ	38.8				Belgium	BE	87.5
Hungary	HU	39.6				Finland	FI	89.0
Greece	GR	41.0				Japan	JP	92.4
Russia	RU	41.6						
Latvia	LV	43.6						
Mean		35.7	*Mean*		61.4	*Mean*		85.5

Notes: RR is the Recovery Rate, defined as the percentage of debt recovered by secured creditors in insolvency proceedings (World Bank Doing Business). Group means are unweighted across countries; the full-sample mean is 64.1. Sample: 22 European countries plus Japan.

within-firm specification, which identifies age effects from within-firm variation over time.

The within-firm estimates are consistent with the baseline findings. Controlling for time-invariant firm characteristics, short-term leverage exhibits a hump-shaped profile, while long-term leverage declines monotonically during the firm's ELC. Log- and total- leverage are increasing after age bin 6-7 consistent with firms starting new investment projects. Magnitudes are similar to the APC estimates. The persistence of these patterns within firms reduces concerns that the baseline profiles are driven primarily by unobserved heterogeneity or compositional selection across cohorts.

Table 7 presents the regression results supporting Figure 4. Table 8 presents the results from a specification that further controls for firm size, measured by worker compensation, in logs. The pattern remains, which constitutes evidence that leverage dynamics relate to firm age, conditionally on firm size differentials.

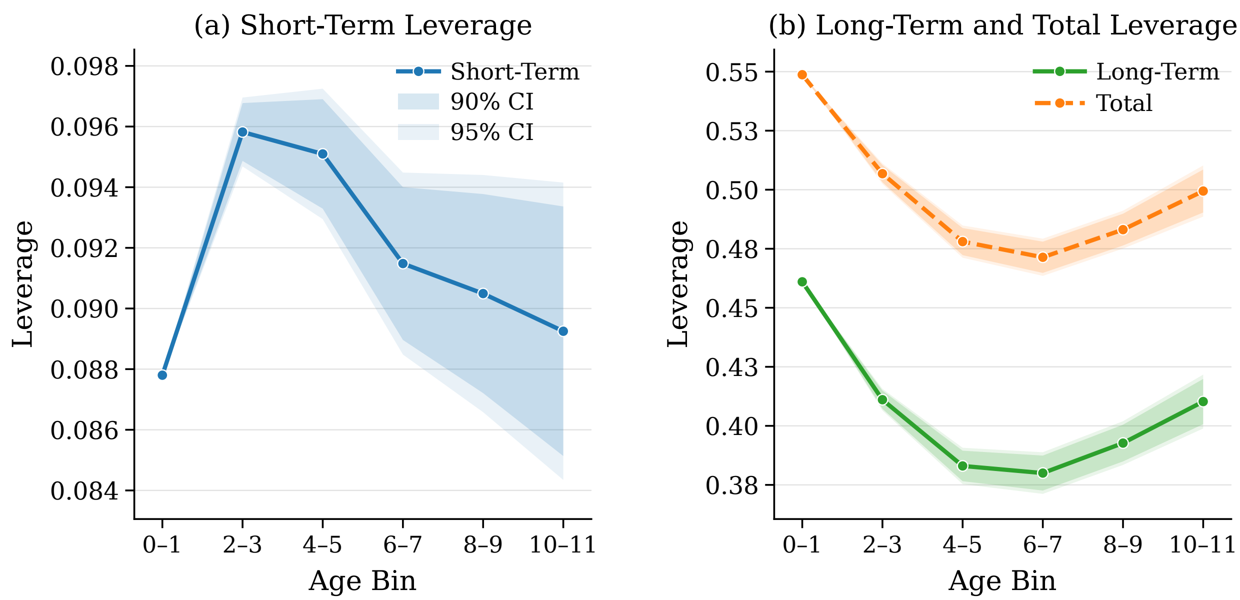

FIGURE 4: CAPITAL STRUCTURE EVOLUTION: WITHIN-FIRM ESTIMATES

Notes: Coefficients from regressions with firm and country×sector×year fixed effects. Age bin 0–1 serves as the reference category; predicted levels are anchored at the unconditional group mean at ages 0–1. Panel (a) plots short-term leverage. Panel (b) plots long-term leverage (solid) and total leverage (dashed). Shaded regions represent 90% and 95% confidence intervals. Standard errors clustered at the country×sector level. Sample restricted to firms with $\text{LTD}_i = 1$, observed for at least six years.

Table 7: Leverage Life-Cycle Profiles for $\text{LTD}_i = 1$ Firms: Within-Firm

	Leverage		
	Short-term (1)	Long-term (2)	Total (3)
A. Unconditional mean (age 0–1)			
Mean level	0.0878	0.4610	0.5487
B. Age effects (relative to age 0–1)			
Age 2–3	0.0080***	−0.0499***	−0.0419***
	(0.0006)	(0.0023)	(0.0022)
Age 4–5	0.0073***	−0.0780***	−0.0707***
	(0.0011)	(0.0039)	(0.0035)
Age 6–7	0.0037**	−0.0810***	−0.0773***
	(0.0015)	(0.0045)	(0.0040)
Age 8–9	0.0027	−0.0683***	−0.0656***
	(0.0020)	(0.0047)	(0.0041)
Age 10–11	0.0014	−0.0507***	−0.0493***
	(0.0025)	(0.0058)	(0.0055)
Country×Sector×Year FE		Yes	
Firm FE		Yes	
Observations	7,109,606	7,109,606	7,109,606
Clusters	1,378	1,378	1,378

Notes: Sample restricted to firms with positive long-term debt at entry ($\text{LTD}_i = 1$). Panel A reports the unconditional mean of each leverage measure among age-bin 0–1 firms in the estimation sample ($N =$ 1,406,303). Panel B reports age-bin coefficients relative to the age 0–1 reference category. Short-term leverage = loans / (LTD + loans + equity); long-term leverage = LTD / (LTD + loans + equity); total leverage = (LTD + loans) / (LTD + loans + equity). Standard errors clustered at the country×sector level. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 8: Leverage Life-Cycle Profiles for $\text{LTD}_i = 1$ Firms: Within-Firm Estimates, Controlling for Compensation

	Leverage		
	Short-term (1)	Long-term (2)	Total (3)
A. Unconditional mean (age 0–1)			
Mean level	0.0902	0.4473	0.5375
B. Age effects (relative to age 0–1)			
Age 2–3	0.0078***	−0.0462***	−0.0384***
	(0.0004)	(0.0019)	(0.0017)
Age 4–5	0.0069***	−0.0700***	−0.0632***
	(0.0008)	(0.0042)	(0.0038)
Age 6–7	0.0022**	−0.0671***	−0.0649***
	(0.0009)	(0.0045)	(0.0044)
Age 8–9	0.0007	−0.0464***	−0.0457***
	(0.0013)	(0.0031)	(0.0034)
Age 10–11	−0.0006	−0.0240***	−0.0246***
	(0.0019)	(0.0043)	(0.0053)
C. Compensation control			
ln(compensation)	0.0065***	0.0050***	0.0115***
	(0.0005)	(0.0010)	(0.0011)
Country×Sector×Year FE		Yes	
Firm FE		Yes	
Observations	4,402,521	4,402,521	4,402,521
Clusters	1,213	1,213	1,213

Notes: Sample restricted to firms with positive long-term debt at entry ($\text{LTD}_i = 1$) and non-missing compensation. Panel A reports the unconditional mean of each leverage measure among age-bin 0–1 firms in the estimation sample (N = 864,856). Panel B reports age-bin coefficients relative to the age 0–1 reference category. Panel C reports the coefficient on log total compensation. Short-term leverage = loans / (LTD + loans + equity); long-term leverage = LTD / (LTD + loans + equity); total leverage = (LTD + loans) / (LTD + loans + equity). Standard errors clustered at the country×sector level. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

A.4 Supporting Tables

Table 9 reports the results from specification (19) in the main text. The results are plotted in Figure 1 in the main text.

TABLE 9: SHORT-TERM LEVERAGE LIFE-CYCLE PROFILES BY INITIAL LONG-TERM DEBT STATUS

	Short-Term Leverage	
	$\text{LTD}_i = 0$	$\text{LTD}_i = 1$
A. Predicted short-term leverage		
Age 0–1	0.0971	0.0878
Age 2–3	0.0962	0.0948***
	(0.0029)	(0.0010)
Age 4–5	0.0967	0.0929***
	(0.0046)	(0.0013)
Age 6–7	0.0974	0.0877
	(0.0056)	(0.0019)
Age 8–9	0.0993	0.0845
	(0.0061)	(0.0020)
Age 10–11	0.1010	0.0807***
	(0.0065)	(0.0021)
Country×Sector×Year FE	Yes	Yes
Observations	16,719,805	7,092,535
Clusters	1,440	1,390

Notes: Each column is estimated from a separate subsample regression of short-term leverage on age-bin indicators, Deaton-normalised cohort controls, and absorbed country×sector×year fixed effects, restricted to firms with ≥ 6 observations over ages 0–11. The predicted level at age 0–1 is the unconditional group mean; predicted levels at subsequent age bins are the unconditional mean plus the estimated age-bin coefficient $\hat{\beta}_k$. Standard errors in parentheses are exact within-group estimates, clustered at the country×sector level; significance refers to the test H_0: $\beta_k = 0$. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 10 reports the results from the specification that focuses on LTD=1 firms only. The results are plotted in Figure 2 in the main text.

Table 11 reports the results from the specification that focuses on LTD=1 firms only in three different country groups, sorted by enfocement levels. The results are plotted in Figure 3 in the main text.

Table 10: Leverage Life-Cycle Profiles for $\text{LTD}_i = 1$ Firms

	Leverage		
	Short-term	Long-term	Total
	(1)	(2)	(3)
A. Unconditional mean (age 0–1)			
Mean level	0.0878	0.4610	0.5487
B. Age effects (relative to age 0–1)			
Age 2–3	0.0071***	-0.0824***	-0.0753***
	(0.0010)	(0.0032)	(0.0037)
Age 4–5	0.0052***	-0.1440***	-0.1388***
	(0.0013)	(0.0066)	(0.0073)
Age 6–7	-0.0001	-0.1796***	-0.1798***
	(0.0019)	(0.0086)	(0.0100)
Age 8–9	-0.0033	-0.2000***	-0.2033***
	(0.0020)	(0.0083)	(0.0097)
Age 10–11	-0.0070***	-0.2146***	-0.2217***
	(0.0021)	(0.0082)	(0.0096)
Country×Sector×Year FE		Yes	
Observations		7,092,535	
Clusters		1,390	

Notes: Sample restricted to firms with positive long-term debt at entry ($\text{LTD}_i = 1$). Panel A reports the unconditional mean leverage at the reference age bin (0–1) for each measure. Panel B reports the change in leverage relative to that reference bin. Short-term leverage = loans / (LTD + loans + equity); long-term leverage = LTD / (LTD + loans + equity); total leverage = (LTD + loans) / (LTD + loans + equity). Estimated via APC regression following the Deaton (1997) normalization, with country×sector×year fixed effects absorbed and cohort dummies included. Standard errors clustered at the country×sector level. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 11: Short-Term Leverage Life-Cycle Profiles by Recovery Rate Group (Relative to Group Mean)

	Short-Term Leverage		
	Low (1)	Medium (2)	High (3)
A. Reference level (age 0–1)			
Unconditional mean	0.1112	0.0846	0.0834
B. Age effects relative to mean (%, $\beta_k/\bar{s}_0 \times 100$)			
Age 2–3	4.58***	11.18***	3.72***
	(1.00)	(1.97)	(1.29)
Age 4–5	7.74***	10.21***	-3.84**
	(1.40)	(2.63)	(1.75)
Age 6–7	7.91***	2.25	-9.82***
	(1.46)	(3.90)	(1.93)
Age 8–9	7.22***	-1.48	-14.53***
	(1.83)	(4.13)	(2.16)
Age 10–11	2.85	-5.08	-20.03***
	(2.16)	(4.20)	(2.66)
C. Adjacent bin Wald tests, Holm–Bonferroni corrected (%)			
Δ(4–5, 2–3)	3.16†	-0.97	-7.56†
	(0.75)	(0.79)	(0.90)
Δ(6–7, 4–5)	0.16	-7.96†	-5.98†
	(0.74)	(1.58)	(0.64)
Δ(8–9, 6–7)	-0.69	-3.73†	-4.71†
	(0.78)	(0.55)	(0.67)
Δ(10–11, 8–9)	-4.36†	-3.60†	-5.51†
	(0.92)	(0.64)	(1.04)
Country×Sector×Year FE		Yes	
Observations	859,765	3,887,074	2,345,696
Clusters	553	326	511

Notes: Sample restricted to $\text{LTD}_i = 1$ firms with ≥ 6 observations during ages 0–11. Columns correspond to country groups by recovery rate: Low ($\text{RR} \leq 46$), Medium ($46 < \text{RR} \leq 76$), High ($\text{RR} > 76$). Panel A reports the unconditional mean of short-term leverage at age 0–1 for each group ($\bar{s}_0$). Panel B reports age effects β_k expressed as a percentage of $\bar{s}_0$; standard errors in parentheses scaled likewise. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$ (unadjusted). Panel C reports Wald tests of the null hypothesis H_0: $\beta_k - \beta_{k-1} = 0$ (no change between adjacent age bins), expressed as a percentage of $\bar{s}_0$. † indicates rejection of H_0 after Holm–Bonferroni correction ($m = 4$, $\alpha = 0.05$) applied within each group; absence of † indicates the two adjacent bins are statistically indistinguishable.

A.5 Wages

Table 12 documents wage, productivity, and unit labor cost differences across enforcement groups. The high-to-low-enforcement ratio of unit labor costs is 1.04, indicating that cross-country compensation differences largely mirror productivity differences rather than shifts in labor shares. This evidence supports the homogeneity-based comparison used in Section 4.4.

TABLE 12: WAGES, PRODUCTIVITY, AND UNIT LABOUR COSTS BY ENFORCEMENT GROUP

	Levels (USD PPP per hour)		Unit Labour	
	Compensation per hour	GDP per hour	Cost (labour share)	N
High enforcement	41.9	72.6	0.587	11
	(5.2)	(14.5)	(0.061)	
Low enforcement	21.1	37.3	0.565	12
	(5.4)	(8.9)	(0.047)	
Ratio (HE / LE)	1.99	1.95	1.038	

Notes: Group means and standard deviations (in parentheses) across countries, 2000–2019. Compensation per hour and GDP per hour are from the Penn World Table 11.0 (`cgdpo`, `emp`, `avh`), expressed in 2021 USD PPP. Unit labour cost is the labour share of GDP (`labsh`). Ireland and Norway have unusually low labour shares (0.45 and 0.50, respectively), likely reflecting multinational profit shifting and oil-sector rents; excluding them raises the HE mean to 0.61 and the HE / LE ratio to 1.08.

A.6 Working-Capital Share Estimation

TABLE 13: WORKING-CAPITAL SHARE: INDUSTRY-REWEIGHTED ESTIMATES

Panel A: Country-level regression (industry-reweighted ϕ)	
Recovery Rate	-0.0105^{***}
	(0.0036)
Constant	1.2521^{***}
	(0.3258)
R^2	0.174
N (countries)	22
Panel B: Predicted $\hat{\phi}$ at selected recovery rates	
Recovery Rate	$\hat{\phi}$
35 (Low Enforcement avg.)	0.885
46 (Low/Medium threshold)	0.770
63 (Medium Enforcement avg.)	0.596
76 (Medium/High threshold)	0.455
85 (High Enforcement avg.)	0.360
Panel C: Group means of reweighted country ϕ	
Low Enforcement (RR $\leq$ 46)	1.006
Medium Enforcement (46 $<$ RR $\leq$ 76)	0.464
High Enforcement (RR $>$ 76)	0.345
Grand mean	0.642

Notes: Dependent variable is the industry-reweighted mean of loans / cost of employees among unconstrained firms (ages 4–11). Country values are constructed by collapsing to country×2-digit NACE sector medians and aggregating with global industry weights (share of total trimmed-sample observations in each sector), holding the industry mix constant across countries. Recovery Rate is the World Bank Doing Business recovery rate (cents on the dollar). Robust standard errors in parentheses. Panel B reports predicted values from the estimated regression. Panel C reports unweighted means of the reweighted country estimates within each enforcement group. $^{***}p < 0.01$. Sample: 22 European countries plus Japan; unconstrained firms defined as age-bin ≥ 3 (ages 4–11), loans > 0, cost of employees > 0, initial_yes $= 1$, $\phi_{\text{ratio}} \leq 10$.

Table 14 reports the estimation of ϕ, the fraction of the wage bill financed through short-term debt, used in Section 4.6 to convert the funding gap into employment units. The working-capital share declines with enforcement quality, which is consistent with firms in high-enforcement economies substituting toward other financing instruments as short-term borrowing constraints ease.

Table 14: Working-Capital Share Estimates

Panel A: Country-level regression

Recovery Rate	-0.0085^{***}
	(0.0018)
Constant	1.0140^{***}
	(0.1192)
R^2	0.539
N (countries)	22

Panel B: Predicted $\hat{\varphi}$ at selected recovery rates

Recovery Rate	$\hat{\varphi}$
35 (Low Enforcement avg.)	0.72
45	0.63
61 (Medium Enforcement avg.)	0.50
75	0.38
85 (High Enforcement avg.)	0.30
90	0.25

Notes: Dependent variable is the country-level median of loans/cost of employees among unconstrained firms (ages 4–11), constructed as an industry- weighted median across 2-digit NACE sectors. Recovery Rate is the World Bank Doing Business recovery rate (cents on the dollar). Robust standard errors in parentheses. Panel B reports predicted values from the estimated regression at selected recovery rates corresponding to enforcement group averages and boundaries. $^{***}p < 0.01$. Sample: 22 European countries.

A.7 Construction of Constraint Estimators

This section documents the sample restrictions, weights, and aggregation procedures used to compute the RM and R-moment estimators.

The starting point is the cleaned ORBIS panel described in Section 2.1. All financial variables are converted to U.S. dollars. We group firm-age observations into two-year age bins (ages 0–1 form age bin 1, ages 2–3 form age bin 2, and so on) and collapse the panel to the firm×age-bin level by averaging within each cell. Each firm contributes at most one observation per age bin.

Cross-country comparison requires a composition adjustment. We construct fixed industry weights $\{w_j\}$ from worldwide operating revenue at the two-digit NACE Rev. 2 level in 2018,

$$w_j = \frac{\text{Rev}^{\text{world}}_{j,2018}}{\sum_{j'} \text{Rev}^{\text{world}}_{j',2018}}.$$

To ensure comparability across enforcement groups and age bins, we restrict the analysis to industries that have at least ten firm observations in every enforcement-group×age-bin cell. Industries that fail this requirement in any cell are dropped. We renormalize weights within the common support,

$$\tilde{w}_j = \frac{w_j}{\sum_{j' \in \mathcal{J}^*} w_{j'}}, \qquad \mathcal{J}^* = \big\{ j : N_{a,j,g} \geq 10 \ \ \forall\, a, g \big\}.$$

After this restriction, 64 two-digit NACE industries remain in each enforcement group, covering over 99% of total sample revenue.

For each enforcement group g, we designate a reference age bin $\hat{a}^g$ corresponding to the final age bin of the estimated early life cycle (ELC): age bin 3 (ages 4–5) for low enforcement and age bin 2 (ages 2–3) for medium and high enforcement. We retain firms that report positive short-term debt at the reference age bin and restrict each firm's observations to age bins at or below the group-specific ELC exit.

The ratio-of-means (RM) estimator is defined as:

$$\Theta_a^{RM,g} = \frac{\sum_j \tilde{w}_j \ \mu_{a,j,g}}{\sum_j \tilde{w}_j \ \mu_{\hat{a},j,g}}, \qquad \mu_{a,j,g} = \frac{1}{N_{a,j,g}} \sum_{i \in (a,j,g)} S_{a,i}.$$

To align numerator and denominator samples, we restrict to firms observed in both age bin a and the reference age bin $\hat{a}$. This balanced restriction ensures that $\Theta_a^{RM,g}$ estimates $\mathbb{E}[S_a \mid j, g]/\mathbb{E}[S_{\hat{a}} \mid j, g]$ within each industry cell, consistent with the population ratio-of-means under random firm exit.

The scale computation uses unconditional means. We estimate the R moment (Section 4.4) at the industry level,

$$R^g = \frac{\sum_{j\in\mathcal{J}^*} \tilde{w}_j \, \mathbb{E}[L_{0,i} \mid j, g]}{\sum_{j\in\mathcal{J}^*} \tilde{w}_j \, \mathbb{E}[S_{\hat{a},i} \mid j, g]}.$$

A.8 Eurostat Comparison

The database tracks each birth cohort of employer enterprises from entry through age five, reporting: `ENT_BIRTH`, the number of enterprises in the birth cohort; `ENT_SRVL`(a), the count of surviving enterprises at age a; and `EMP_SRVL`(a), total employment of the survivors. We use annual cohorts from 2004–2014. For each enforcement group c and age bin b, we set

$$n_b^c \;=\; \frac{\texttt{EMP\_SRVL}(b)}{\texttt{ENT\_BIRTH}},$$

GDP-weighted across countries in the group. Using `ENT_BIRTH` in the denominator ensures that n_b^c captures both the intensive margin (employment per surviving firm) and the extensive margin (differential exit across enforcement groups); conditioning on survivors only would remove the exit margin and understate the total gap (Criscuolo et al., 2014). The aggregate Eurostat employment gap is

$$\hat{\mathcal{E}} \;=\; \sum_b \tilde{\omega}_b \left[1 - \frac{n_b^c}{n_b^{HE}}\right], \tag{20}$$

where $\tilde{\omega}_b \;=\; \left[\sum_{a\in b}(1-\rho)\rho^a\right] / \left[\sum_{a=0}^{5}(1-\rho)\rho^a\right]$ are the same stationary weights as in the funding gap index. With $\rho = 0.90$ the bin weights are $(\tilde{\omega}_1, \tilde{\omega}_2, \tilde{\omega}_3) = (0.406,\ 0.329,\ 0.266)$ for bins 0–1, 2–3, and 4–5.

Country-level diagnostics reveal two composition effects that bear on the comparison. In the LE group, Greece exhibits employment-per-entrant ratios above unity at all age bins, generating a negative country-level gap of 40 percent. This reflects positive selection into formal-sector entry: high registration costs in Greece deter small, marginal entrants while leaving larger, better-capitalized firms in the formal sector (Djankov et al., 2002).[11] This entry-cost selection is distinct from enforcement: it biases the GDP-weighted LE/HE ratio toward one and causes the raw gap to *understate* the enforcement effect. In the ME group,

[11] A leave-one-out exercise confirms that Greece alone accounts for −13.3 percentage points of the GDP-weighted LE gap; no other LE country shifts the aggregate by more than 2.5 percentage points. Latvia and Croatia exhibit ratios modestly above unity at some age bins for the same reason but carry smaller GDP weights.

France (29 percent of ME GDP) exhibits a country-level gap of 47.7 percent, driven by employment concentration below the statutory 50-employee threshold (Garicano et al., 2016). This threshold effect is similarly unrelated to contract enforcement quality.

We therefore report the Eurostat comparison for samples that exclude these composition outliers: the LE group excludes Greece (CZ, EE, HR, HU, LV remaining) and the ME group excludes France (ES, IT, PL, PT remaining). The HE benchmark comprises SE, DE, BE, and FI throughout; Ireland is excluded because measured GDP per capita is inflated by multinational profit shifting.[12]

[12]Including Ireland in HE reduces the measured HE employment level, compressing the LE and ME ratios toward unity. With Ireland, the LE (excl. Greece) gap falls from 27.6 to 24.1 percent; the model comparison is qualitatively unchanged.

Online Appendix

Contract Enforcement and Young Firm Capital Structure: A Global Perspective

Gonzalo Basante   Ina Simonovska

Not for publication

Contents

OA.1 Data Coverage and Sample Selection

This appendix details the construction of our firm-level panel and justifies key sample selection decisions. We characterize corporate capital structure and leverage dynamics over the life cycle of young firms across countries with varying levels of financial development, requiring careful attention to data coverage and firm heterogeneity.

OA.1.1 Country Coverage and Panel Structure

We use the ORBIS historical series provided by the NBER, and we begin with the cleaned version of ORBIS constructed following Kalemli-Özcan et al. (2024), applying standard protocols to eliminate duplicate records, harmonize firm identifiers, and exclude observations with implausible financial data. These procedures are now standard in the firm dynamics literature using ORBIS (Gopinath et al., 2017).

We focus on young firms, which we define to be of ages 11 and below, during the 1998-2021 period. Tables OA.1 and OA.2 report coverage metrics across countries: the number of firms, average years observed per firm, and the share of firms with at least 3, 6, and 9 years of data. These metrics provide a screening mechanism for determining whether a country has sufficient longitudinal depth to support life-cycle analysis.

The substantial cross-country variation in both firm counts and panel quality creates a natural tension in sample construction. Countries with the largest firm populations (Russia, France, United Kingdom) do not necessarily provide the deepest panel coverage, while smaller economies often exhibit superior data quality. This variation reflects differences in reporting requirements, coverage policies across Bureau van Dijk's data collection procedures, and the underlying business environment.

OA.1.2 Cross-Country Variation in Institutional Quality

The cross-country variation in firm financing patterns provides crucial identification for our enforcement-based theory. Tables OA.3 and OA.4 show that the percentage of firms with positive initial long-term debt ranges from 0.4% in India to 34.5% in Belgium, providing substantial variation for identifying the effects of institutional quality on financing patterns.

This variation is systematically related to institutional development. Table OA.5 confirms that countries with higher recovery rates and GDP per capita exhibit significantly higher prevalence of LTD $= 1$ firms, consistent with our theoretical prediction that stronger enforcement institutions facilitate long-term debt financing.

Table OA.1: Country Coverage in NBER Historical ORBIS Data (Part 1)

Country	Firms (000s)	Avg Years per Firm	Panel Coverage (%) 3 years	6 years	9 years	Included in Sample
Russia	3002	4.1	62	28	9	Yes
France	1557	4.4	61	32	15	Yes
United Kingdom	1498	3.5	49	21	9	Yes
Italy	1249	5.1	70	40	21	Yes
China	1069	2.4	37	5	0	Yes
Spain	786	4.5	64	32	14	Yes
Ukraine	492	5.2	72	43	21	Yes
Germany	476	3.3	51	18	5	Yes
Sweden	469	5.0	76	42	13	Yes
Belgium	460	5.8	74	48	29	Yes
Bulgaria	426	4.5	65	34	14	Yes
South Korea	407	3.7	57	22	8	Yes
Norway	351	4.2	65	29	11	Yes
Portugal	312	3.5	51	21	7	Yes
Romania	245	3.9	62	29	5	Yes
Serbia	194	4.4	69	31	13	Yes
Hungary	189	3.2	49	17	5	Yes
Czech Republic	163	3.6	55	21	7	Yes
Finland	162	3.6	56	23	6	Yes
Slovakia	162	4.3	63	30	13	Yes
Croatia	153	5.0	73	40	16	Yes
Morocco	137	3.2	48	18	3	Yes
Denmark	125	3.2	51	18	4	Yes
Colombia	121	2.7	39	10	2	Yes
Japan	119	3.2	51	15	4	Yes
Latvia	102	4.1	60	29	11	Yes
Poland	96	3.3	49	18	5	Yes
Austria	72	3.0	50	11	3	Yes

Notes: This table presents country coverage statistics from the NBER Historical ORBIS database following the data cleaning methodology of Kalemli-Özcan et al. (2024) for firms aged 11 or below. Countries are ranked by number of unique firms. Firms (000s) shows thousands of unique firms per country. Avg Years per Firm shows mean years of panel data per firm. Panel Coverage columns show the percentage of firms with at least 3, 6, and 9 years of consecutive data, respectively. Our final sample includes countries with at least 10,000 unique firms, providing sufficient statistical power for analyzing young firm dynamics over their early life cycle.

Table OA.2: Country Coverage in NBER Historical ORBIS Data (Part 2)

Country	Firms (000s)	Avg Years per Firm	Panel Coverage (%) 3 years	6 years	9 years	Included in Sample
Estonia	66	5.4	74	46	22	Yes
Slovenia	64	2.6	28	10	3	Yes
Luxembourg	55	3.1	50	14	1	Yes
Ireland	48	4.0	61	26	10	Yes
India	45	4.7	78	40	2	Yes
Netherlands	41	2.0	30	1	0	Yes
Singapore	41	2.5	40	6	1	Yes
Greece	40	4.8	65	35	18	Yes
Turkey	36	3.2	55	13	3	Yes
Malaysia	34	3.0	47	11	2	Yes
Iceland	28	4.1	62	27	9	Yes
Philippines	20	3.6	53	25	7	Yes
Algeria	17	2.1	30	5	0	Yes
Bosnia and Herzegovina	16	6.9	87	60	36	Yes
Brazil	11	2.6	36	7	1	Yes
Montenegro	9	3.1	46	20	1	No
Australia	9	3.3	52	16	3	No
Mexico	6	1.7	14	1	0	No
Lithuania	6	3.3	48	17	6	No
Malta	6	4.8	69	38	16	No
Taiwan	5	3.1	45	15	5	No
Belarus	2	1.7	5	1	0	No
Kazakhstan	1	4.1	65	29	7	No
Included (46 countries)	19449		61	34	19	
Excluded (5 countries)	30					

Notes: This table presents country coverage statistics from the NBER Historical ORBIS database following the data cleaning methodology of Kalemli-Özcan et al. (2024) for firms aged 11 or below. Countries are ranked by number of unique firms. Firms (000s) shows thousands of unique firms per country. Avg Years per Firm shows mean years of panel data per firm. Panel Coverage columns show the percentage of firms with at least 3, 6, and 9 years of consecutive data, respectively. Our final sample includes countries with at least 10,000 unique firms, providing sufficient statistical power for analyzing young firm dynamics over their early life cycle.

Table OA.3: Cross-Country Sample Coverage: Countries with Highest LTD=1 Firm Counts

Country	Total Firms (000s)	LTD=1 Firms (000s)	% LTD=1	% LTD=1 w/ 3+ yrs	% LTD=1 w/ 6+ yrs	% LTD=1 w/ 9+ yrs	Recovery Rate	GDP p.c. (000s USD)
France	1741.3	585.9	33.6	65.8	37.3	20.4	55.5	35.7
Spain	910.3	310.7	34.1	71.5	44.1	28.6	73.6	25.9
Russia	3210.6	257.8	8.0	62.7	26.3	10.9	41.6	9.3
Belgium	538.7	186.0	34.5	81.3	60.5	45.2	87.6	39.8
Korea	468.9	155.6	33.2	59.4	27.5	13.3	82.9	27.2
Italy	1470.4	155.1	10.5	79.9	52.8	33.7	61.7	31.9
United Kingdom	1695.3	149.8	8.8	56.0	30.7	19.8	86.6	43.8
Sweden	522.7	99.9	19.1	85.3	63.6	43.8	76.6	49.8
Germany	682.0	94.1	13.8	61.0	29.5	13.3	81.6	40.4
Portugal	387.3	93.7	24.2	63.5	33.4	17.4	69.1	19.7
Norway	370.6	43.3	11.7	72.2	44.1	26.5	91.1	74.8
Finland	210.4	34.7	16.5	71.2	43.4	25.3	89.0	43.4
China	1436.7	29.6	2.1	54.7	10.8	2.2	36.6	6.8
Croatia	160.8	25.9	16.1	75.0	43.2	25.1	30.9	12.5
Japan	314.7	24.6	7.8	63.0	31.1	17.9	92.4	34.2
Poland	119.7	18.7	15.6	56.4	26.0	13.5	48.9	12.4
Czech Republic	179.4	18.7	10.4	63.3	33.3	19.2	42.7	16.9
Estonia	66.4	18.6	28.1	81.4	55.8	36.5	38.7	16.5
Denmark	161.4	18.3	11.3	66.2	28.8	8.5	87.8	55.2
Bulgaria	484.1	17.2	3.6	82.1	54.0	34.0	34.4	7.2
Serbia	246.6	15.8	6.4	76.3	45.4	29.8	28.1	5.9
Latvia	110.4	14.1	12.7	67.9	38.4	19.6	43.6	13.2
Ireland	61.7	10.7	17.3	70.8	41.0	21.6	87.0	61.6
Ukraine	607.7	10.4	1.7	65.5	24.3	10.8	9.4	2.2
Slovenia	94.7	8.6	9.0	65.2	30.8	13.2	67.4	22.4

Notes: This table shows the countries with the highest number of firms with initial long-term debt (LTD=1). Countries are ordered by number of LTD=1 firms. Total Firms shows the number of firms (in thousands) in our sample. LTD=1 Firms shows firms with positive long-term debt at ages 0-1. % LTD=1 is the percentage of all firms that are LTD=1. The next three columns show the percentage of LTD=1 firms observed for 3+, 6+, and 9+ years respectively. Recovery Rate is the percentage recovery in insolvency proceedings (World Bank Doing Business). GDP p.c. is GDP per capita in thousands of 2015 USD.

Table OA.4: Cross-Country Sample Coverage: Countries with Lower LTD=1 Firm Counts

Country	Total Firms (000s)	LTD=1 Firms (000s)	% LTD=1	% LTD=1 w/ 3+ yrs	% LTD=1 w/ 6+ yrs	% LTD=1 w/ 9+ yrs	Recovery Rate	GDP p.c. (000s USD)
Austria	102.0	8.4	8.2	45.2	17.2	6.6	75.2	43.4
Morocco	151.2	8.3	5.5	47.7	18.7	3.2	27.8	3.1
Slovakia	171.6	6.8	3.9	74.9	43.5	26.7	50.9	16.1
Iceland	30.5	6.7	22.1	72.7	45.5	28.0	82.2	51.4
India	1606.3	6.5	0.4	84.2	47.5	4.3	32.9	1.9
Greece	52.5	4.3	8.3	74.6	54.5	41.0	40.1	19.9
Turkey	56.0	4.0	7.1	63.7	21.1	7.3	19.4	10.5
Algeria	19.4	3.8	19.4	41.4	6.6	0.0	50.8	4.6
Hungary	211.4	3.5	1.6	68.3	40.5	22.8	39.9	11.8
Bosnia & Herzegovina	30.4	2.8	9.3	94.2	76.0	54.3	35.9	4.3
Singapore	53.0	2.6	4.9	49.8	13.5	2.2	88.9	56.7
Netherlands	81.0	2.6	3.2	24.0	4.8	0.9	86.9	40.5
Luxembourg	63.5	2.0	3.1	64.3	27.2	10.2	43.6	106.7
Australia	18.5	1.9	10.4	58.8	21.0	5.6	82.0	57.2
Romania	247.0	1.7	0.7	86.2	59.2	20.7	15.5	5.8
Colombia	158.4	1.7	1.1	60.5	25.4	11.3	59.9	5.4
Malaysia	64.2	1.3	2.0	71.1	24.3	6.2	75.3	10.1
Lithuania	7.7	1.2	15.7	55.9	31.7	20.9	44.5	13.0
Malta	6.9	1.1	16.5	74.0	48.6	28.7	39.4	23.6
Brazil	21.5	0.9	4.0	51.2	15.3	2.6	17.4	8.8
Montenegro	10.4	0.6	5.3	71.7	38.1	4.3	49.0	6.8
Philippines	34.1	0.4	1.0	65.2	45.0	26.3	14.3	2.8
Belarus	4.4	0.3	6.7	3.7	0.0	0.0	38.4	6.3
Mexico	13.5	0.2	1.5	33.2	7.0	3.0	67.3	9.9

Notes: This table shows the countries with lower numbers of firms with initial long-term debt (LTD=1). Countries are ordered by number of LTD=1 firms. Total Firms shows the number of firms (in thousands) in our sample. LTD=1 Firms shows firms with positive long-term debt at ages 0-1. % LTD=1 is the percentage of all firms that are LTD=1. The next three columns show the percentage of LTD=1 firms observed for 3+, 6+, and 9+ years respectively. Recovery Rate is the percentage recovery in insolvency proceedings (World Bank Doing Business). GDP p.c. is GDP per capita in thousands of 2015 USD.

Table OA.5: LTD=1 Firm Prevalence and Institutional Quality

	(1) Recovery Rate	(2) ln GDP p.c.	(3) Both
Recovery Rate	0.145^{**}		0.052
	(0.052)		(0.081)
ln GDP per capita		3.935^{***}	2.954
		(1.250)	(1.986)
R-squared	0.142	0.174	0.181
Observations	45	45	45

Notes: Dependent variable is the percentage of firms with initial long-term debt (% LTD=1). Recovery Rate is from World Bank Doing Business indicators. Standard errors in parentheses. Sample includes countries with at least 100 LTD=1 firms.

The positive coefficients on both recovery rates (0.145, significant at 5%) and log GDP per capita (3.935, significant at 1%) suggest that a one-standard-deviation improvement in institutional quality is associated with approximately 5-8 percentage points higher LTD = 1 prevalence.

OA.1.3 Sample Construction for Main Analysis

Based on the above analysis, our main empirical work focuses on the LTD = 1 population, where financial constraints are more likely to govern investment and growth decisions rather than heterogeneous entrepreneurial preferences. This sample restriction ensures that observed capital structure dynamics reflect financing considerations, thereby providing a cleaner test of our theoretical predictions about enforcement constraints and firm financial behavior.

For country inclusion, we require two criteria: (1) at least 100 LTD = 1 firms to ensure statistical power, and (2) at least 30% of LTD = 1 firms observed for 6+ years to ensure adequate panel quality for studying financing dynamics. This yields a sample spanning the full range of institutional quality while maintaining sufficient data depth to trace firm capital structure evolution over the early life cycle.

Figures OA.1 and OA.2 illustrate this sample construction strategy. The tertile approach reveals natural breakpoints in the data while maintaining balanced representation across development levels.

Our final sample comprise firms from countries meeting both coverage and quality thresholds, spanning the full range of institutional development while maintaining adequate statistical power to identify the causal effects of enforcement quality on young firm capital structure dynamics. This careful sample construction underpins the credibility of our empirical findings and strengthens the external validity of our theoretical framework.

OA.1.4 Cohort coverage and anchor selection

As described in the main text, we implement a Deaton-style Age-Period-Cohort (APC) framework to estimate the leverage lifecycle patterns of young firms across countries, exploiting variation by firm age (grouped into two-year bins), period, and cohort of incorporation. Below we describe our country selection procedure in order to carry out the analysis.

Table OA.6 reports the number of firms by country and cohort for 1996–2000. We require each country to have at least 200 firms in both 1998 and 1999 cohorts to ensure

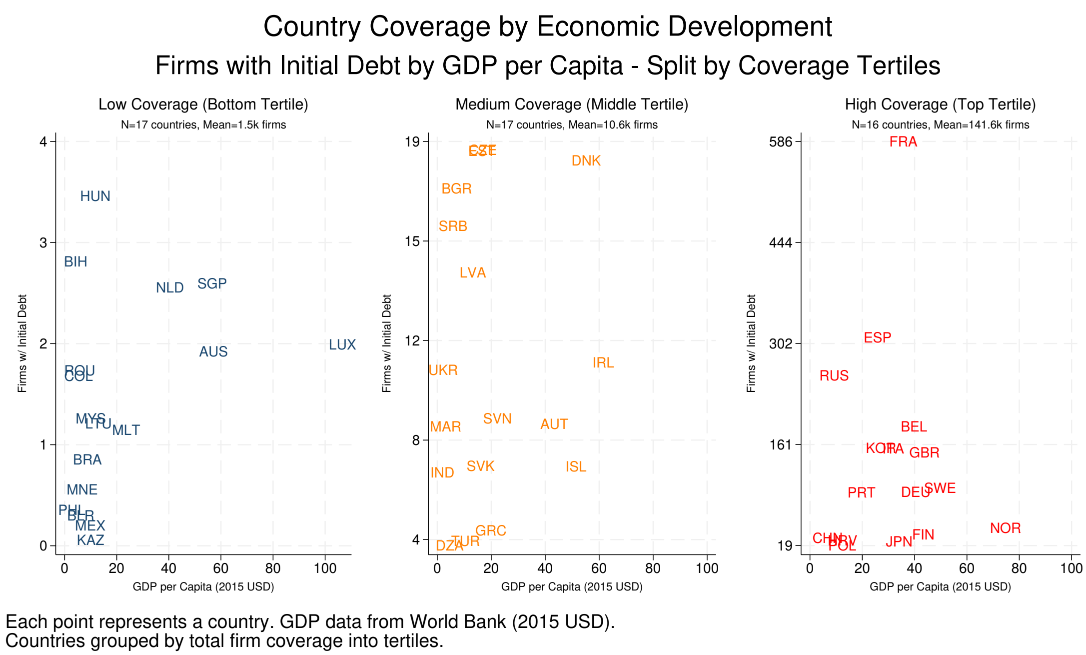

Figure OA.1: Country Coverage by Economic Development: Total LTD=1 Firms by Coverage Tertiles

Notes: Countries are grouped into tertiles based on total number of firms with initial long-term debt. Each subplot uses a continuous Y-axis scale where the middle tertile begins at the maximum of the bottom tertile, and the top tertile begins at the maximum of the middle tertile. This scaling facilitates identification of natural breakpoints for sample construction. GDP per capita data from World Bank (2015 USD).

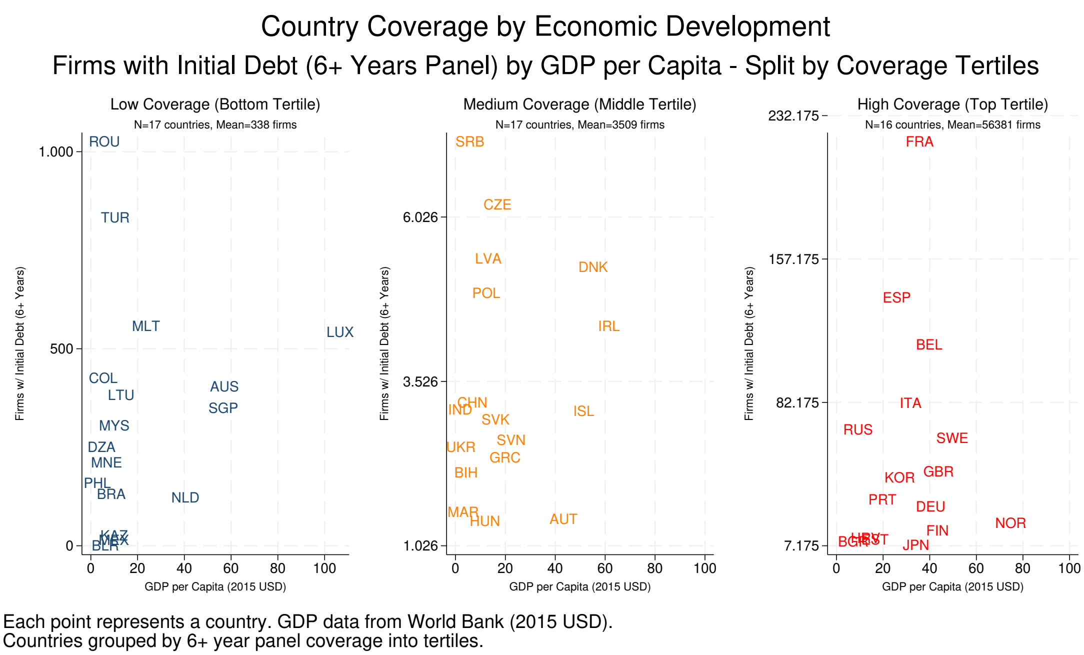

Figure OA.2: Country Coverage by Economic Development: LTD=1 Firms with 6+ Years Panel Data by Coverage Tertiles

Notes: Countries are grouped into tertiles based on number of LTD=1 firms observed for six or more years, demonstrating the trade-off between sample size and panel quality. Continuous Y-axis scaling across tertiles shows the substantial reduction in coverage when requiring adequate panel depth for life-cycle analysis. GDP per capita data from World Bank (2015 USD).

stable normalization around our anchor cohorts. Countries failing this are excluded from the baseline APC analysis. We drop Serbia from the analysis due to large outliers. This leaves us with a total of 23 countries that carry the Yes label in the table below.

Table OA.6: Firms by Country and Cohort (1996–2000)

Country	1996	1997	1998	1999	2000	Total	Anchor OK
Austria	12	6	11	7	88	124	No
Belgium	31,929	46,803	55,839	61,538	72,499	268,608	Yes
Bulgaria	68	101	116	938	1,385	2,608	No
Bosnia & Herz.		195	479	1,220	1,983	3,877	Yes
China			14	12	10	36	No
Czech Rep.	710	941	966	1,155	2,589	6,361	Yes
Germany	102	322	535	1,212	1,999	4,170	Yes
Denmark			16	28	34	78	No
Spain	57,784	82,156	90,488	100,104	105,710	436,242	Yes
Estonia	1,358	3,227	3,503	3,961	5,603	17,652	Yes
Finland	5,426	7,766	6,577	6,230	7,941	33,940	Yes
France	32,785	56,892	61,629	66,779	70,175	288,260	Yes
UK	14,573	27,956	33,555	36,025	34,061	146,170	Yes
Greece	1,175	1,775	1,371	1,892	2,177	8,390	Yes
Croatia	254	653	644	864	3,575	5,990	Yes
Hungary	293	797	713	514	288	2,605	Yes
India					1	1	No
Ireland	1	88	870	2,552	2,797	6,308	Yes
Iceland	1,235	1,951	1,805	1,908	1,904	8,803	Yes
Italy	772	1,243	1,694	2,735	7,720	14,164	Yes
Japan	661	889	1,220	1,584	2,994	7,348	Yes
S. Korea	186	393	1,811	21,098	34,641	58,129	No
Latvia	613	979	859	742	1,081	4,274	Yes
Poland	490	828	1,178	1,642	1,983	6,121	Yes
Portugal	1,327	3,197	3,650	4,001	5,437	17,612	Yes
Romania	1,116	1,555	1,499	1,829	1,968	7,967	Yes
Russia	436	4,088	6,084	7,623	10,260	28,491	Yes
Serbia	61	1,660	3,231	2,919	4,943	12,814	No
Slovakia	133	121	94	195	238	781	No
Sweden	8,648	17,060	17,551	22,653	24,835	90,747	Yes
Turkey	1		1	1	3	6	No
Ukraine	276	340	1,570	2,648	1,415	6,249	Yes
Total	162,425	263,982	299,573	356,609	412,337	1,494,926	

Notes: Number of firms by cohort (incorporation year, 1996–2000) in our final sample of firms with initial long-term debt (ages 0 or 1). We require each country to have at least 1,000 firms observed four times between ages 0–5, and at least 200 firms in both anchor cohorts (1998 and 1999) to ensure reliable cohort normalization.

OA.2 Theory: Proofs of Propositions

This appendix provides complete proofs for all propositions stated in Sections 3 and 4. The stochastic model is developed formally in Appendix OA.3; proofs relying on its recursive structure reference results established there.

OA.2.1 Proofs of Propositions

OA.2.1.1 Proof of Proposition 1

We compare the present value of costs across financing arrangements.

For intra-period funding needs, a sequence of single-period loans costs $\kappa_C + (1+r_f)S_a$ each period, with present value

$$\sum_{t=0}^{\infty}(\beta\rho)^t[\kappa_C + (1+r_f)S_a] = \frac{\kappa_C}{1-\beta\rho} + \frac{(1+r_f)S_a}{1-\beta\rho}.$$

A multi-period loan covering the same recurring needs incurs a single transaction cost κ_C but charges a premium for exit risk:

$$\kappa_C + (1+r_f)\sum_{t=0}^{\infty}(\beta\rho)^t\frac{S_a}{\rho} = \kappa_C + \frac{(1+r_f)S_a}{\rho(1-\beta\rho)}.$$

The cost difference is

$$\frac{\beta\rho\kappa_C}{1-\beta\rho} - \frac{(1-\rho)(1+r_f)S_a}{\rho(1-\beta\rho)}.$$

By Assumption 2, $\kappa_C < S_0(K(\xi_L), z_L)\frac{1-\rho}{\rho}$, so the transaction cost savings do not outweigh the exit risk premium, making single-period loans optimal for working capital.

For capital acquisition, repayment requires multiple periods (Assumption 3). A multi-period loan incurs a single transaction cost κ_C, while a sequence of T single-period loans incurs $\sum_{t=0}^{T-1}(\beta\rho)^t\kappa_C$. Since $T > 1$, the multi-period loan dominates.

Proof of Proposition 2

The firm chooses $\{q_a, S_a, K\}_{a=0}^{\infty}$ to maximize

$$\max_{\{q_a,S_a,K\}_{a=0}^{\infty}} \sum_{a=0}^{\infty}(\beta\rho)^a \left(z\left[K^{\alpha}\left(\frac{S_a}{w}\right)^{1-\alpha}\right]^{\eta} - (1+r_f)S_a - q_a \right)$$

subject to

$$(1+r_f)p_k K \leq \sum_{a=0}^{\infty}(\beta\rho)^a q_a \tag{PCF}$$

$$0 \leq q_a \leq z\left[K^{\alpha}\left(\frac{S_a}{w}\right)^{1-\alpha}\right]^{\eta} - (1+r_f)S_a \tag{NNC}$$

$$(1-\xi)z\left[K^{\alpha}\left(\frac{S_a}{w}\right)^{1-\alpha}\right]^{\eta} \leq V_a \tag{LE}$$

where $V_a = \sum_{j=0}^{\infty}(\beta\rho)^j \left(z\left[K^{\alpha}\left(\frac{S_{a+j}}{w}\right)^{1-\alpha}\right]^{\eta} - (1+r_f)S_{a+j} - q_{a+j} \right)$ is the continuation value at age a.

Let λ denote the multiplier on (PCF), μ_a^H and μ_a^L the multipliers on the upper and lower bounds of (NNC), and ψ_a the multiplier on (LE). The first-order conditions are

$$\partial q_a \ : \ -(\beta\rho)^a + \lambda(\beta\rho)^a + \mu_a^L - \mu_a^H - \sum_{j=0}^{a}(\beta\rho)^j \psi_{a-j} = 0 \tag{FOC-q}$$

$$\begin{aligned} \partial S_a \ : \ & \left[(\beta\rho)^a + \mu_a^H + \sum_{j=0}^{a}(\beta\rho)^j \psi_{a-j}\right]\left[\frac{\eta(1-\alpha)}{S_a} z\left[K^{\alpha}\left(\frac{S_a}{w}\right)^{1-\alpha}\right]^{\eta} - (1+r_f)\right] \\ & = \psi_a(1-\xi)\frac{\eta(1-\alpha)}{S_a} z\left[K^{\alpha}\left(\frac{S_a}{w}\right)^{1-\alpha}\right]^{\eta} \end{aligned} \tag{FOC-R}$$

Define the unconstrained short-term debt level:

$$S_u(K,z) \equiv \left[\frac{\eta(1-\alpha)}{(1+r_f)} z\left[K^{\alpha}\left(\frac{1}{w}\right)^{1-\alpha}\right]^{\eta}\right]^{\frac{1}{1-\eta(1-\alpha)}}.$$

From (FOC-R), $\psi_a = 0$ if and only if $S_a = S_u(K,z)$.

From (FOC-q), isolating λ:

$$\lambda = 1 + \frac{\mu_a^H - \mu_a^L}{(\beta\rho)^a} + \sum_{j=0}^{a} \psi_{a-j} \left(\frac{1}{\beta\rho}\right)^{a-j} \qquad \forall a.$$

Since λ is constant, comparing periods a and $a+1$ yields

$$\beta\rho(\mu_a^H - \mu_a^L) - \psi_{a+1} = \mu_{a+1}^H - \mu_{a+1}^L.$$

Applying this recursively:

$$(\beta\rho)^a \mu_0^H - \sum_{j=1}^{a} (\beta\rho)^{a-j} \psi_j = \mu_a^H - \mu_a^L.$$

The following claims characterize the dynamics of the optimal contract.

Claim 1 *In a solution with $K > 0$, $\mu_0^H > 0$.*

Proof. Suppose $\mu_0^H = 0$. From (FOC-q) at $a = 0$: $\lambda = 1 - \mu_0^L + \psi_0$, with $\mu_0^L \geq 0$ and $\psi_0 \geq 0$.

Case 1: If $\mu_0^L > 0$, then $q_0 = 0$ by complementary slackness. The recursive relationship with $\mu_0^H = 0$ gives $\mu_a^H - \mu_a^L \leq 0$ for all $a \geq 1$ (since all $\psi_j \geq 0$), implying either $q_a = 0$ for all a or q_a below its upper bound. The participation constraint cannot be satisfied for $K > 0$.

Case 2: If $\mu_0^L = 0$, the recursive relationship gives $\mu_a^H - \mu_a^L = -\sum_{j=1}^{a} (\beta\rho)^{a-j} \psi_j \leq 0$ for all $a \geq 1$. The upper bound on coupons never binds, yielding $q_a = 0$ for all a, which contradicts (PCF) for $K > 0$. ■

Claim 2 *In a solution with $K > 0$, there exists a finite age $\hat{a}$ such that*

$$(\beta\rho)^{\hat{a}} \mu_0^H \leq \sum_{j=1}^{\hat{a}} (\beta\rho)^{\hat{a}-j} \psi_j.$$

Proof. Suppose that $(\beta\rho)^a \mu_0^H > \sum_{j=1}^{a} (\beta\rho)^{a-j} \psi_j$ for all a. Then $\mu_a^H > 0$ for all a, so $q_a = \pi_a$ for all a. Zero dividends imply $V_a = 0$, so (LE) requires $S_a = 0$ and consequently $\pi_a = q_a = 0$, contradicting (PCF) for $K > 0$. ■

Claim 3 *$\mu_a^H - \mu_a^L \leq 0$ for all $a \geq \hat{a}$.*

Proof. At $a = \hat{a}$, the recursive relationship and Claim 2 give $\mu_{\hat{a}}^H - \mu_{\hat{a}}^L \leq 0$. For the induction step, $\mu_{a+1}^H - \mu_{a+1}^L = \beta\rho(\mu_a^H - \mu_a^L) - \psi_{a+1}$. If $\mu_a^H - \mu_a^L \leq 0$ and $\psi_{a+1} \geq 0$, then $\mu_{a+1}^H - \mu_{a+1}^L \leq 0$. ■

Claim 4 $\psi_a = 0$ *for* $a \geq \hat{a}$.

Proof. From Claim 3, $\mu_a^H = 0$ for all $a \geq \hat{a}$. Suppose $\psi_a > 0$ for some $a \geq \hat{a}$. Then $S_a < S_u(K, z)$, but $\mu_a^H = 0$ means the firm can pay positive dividends. Since $d\pi/dS > 0$ for $S_a < S_u(K, z)$, the firm would strictly benefit from redirecting dividend funds to increase S_a toward $S_u(K, z)$, contradicting optimality. Hence $\psi_a = 0$ for all $a \geq \hat{a}$. ■

Claim 5 *For any* $K > 0$*, a coupon structure that repays debt as quickly as possible,*

$$q_a = \begin{cases} \pi_a(K) & \text{if } a < T(K) \\ \in [0, \pi_a(K)] & \text{if } a = T(K) \\ 0 & \text{if } a > T(K) \end{cases}$$

is a solution, where $T(K) \geq \hat{a}$*. Any other* q_a *schedule after* $\hat{a}$ *satisfying (PCF) is also optimal.*

Proof. For $a < \hat{a}$, $\mu_a^H > 0$ by Claims 1–3, so $q_a = \pi_a(K)$. For $a \geq \hat{a}$, $\mu_a^H = 0$ and $\psi_a = 0$, so the firm is indifferent between coupons and dividends. The front-loaded structure sets $q_a = \pi_a(K)$ for $a < T(K)$, where $T(K)$ satisfies

$$(1 + r_f)p_k K = \sum_{a=0}^{T(K)-1} (\beta\rho)^a \pi_a(K) + (\beta\rho)^{T(K)} q_{T(K)},$$

and $q_a = 0$ for $a > T(K)$. Once enforcement ceases to bind (after $\hat{a}$), the timing of coupon versus dividend payments does not affect firm value, so any (PCF)-satisfying schedule is equally optimal. ■

Claim 6 *The optimal dividend structure is*

$$d_a = \begin{cases} 0 & \text{if } a < T(K) \\ \in [0, \pi_u(K)] & \text{if } a = T(K) \\ \pi_u(K) & \text{if } a > T(K). \end{cases}$$

Proof. Since $d_a = \pi_a(K) - q_a$, the result follows from Claim 5. For $a > T(K)$, $q_a = 0$ and $S_a = S_u(K,z)$ (by Claim 4), so $d_a = \pi_u(K)$. ■

Claim 7 *The optimal short-term debt structure is*

$$S_a(K) = \begin{cases} \Theta_a(\gamma) \cdot S_u(K,z) & \text{if } a < \hat{a} \\ S_u(K,z) & \text{if } a \geq \hat{a} \end{cases}$$

where the constraint multiplier is

$$\Theta_a(\gamma) \equiv \left[\beta\rho^{T(K)-a} \cdot \frac{\beta\rho + (1-\beta\rho)(1-\gamma)}{(1-\beta\rho)(1-\xi)} \cdot (1-\eta(1-\alpha))\right]^{\frac{1}{\eta(1-\alpha)}}$$

and $S_u(K,z)$ is the unconstrained level:

$$S_u(K,z) = w\left[\frac{z\eta(1-\alpha)}{w(1+r_f)}\right]^{\frac{1}{1-\eta(1-\alpha)}} K^{\frac{\eta\alpha}{1-\eta(1-\alpha)}}.$$

Proof. For $a \geq \hat{a}$, $\psi_a = 0$ implies $S_a = S_u(K,z)$ by Claim 4. For $a < \hat{a}$, the enforcement constraint binds: $(1-\xi)z[K^\alpha(S_a/w)^{1-\alpha}]^\eta = V_a$. Since $q_a = \pi_a$ for $a < T(K)$ (Claim 5), dividends are zero and

$$V_a = \sum_{j=T(K)-a}^{\infty} (\beta\rho)^j \pi_u(K) = \frac{(\beta\rho)^{T(K)-a}}{1-\beta\rho} \cdot \pi_u(K).$$

Substituting $\pi_u(K) = [1-\eta(1-\alpha)] \cdot \frac{1+r_f}{\eta(1-\alpha)} \cdot S_u(K,z)$ into the binding enforcement constraint and simplifying:

$$\left(\frac{S_a}{S_u(K,z)}\right)^{\eta(1-\alpha)} = \beta\rho^{T(K)-a} \cdot \frac{\beta\rho + (1-\beta\rho)(1-\gamma)}{(1-\beta\rho)(1-\xi)} \cdot (1-\eta(1-\alpha)).$$

Raising to the power $1/\eta(1-\alpha)$ yields $S_a = \Theta_a(\gamma) \cdot S_u(K,z)$. ■

Claim 8 *Employment mirrors short-term debt: $n_a = S_a(K)/w$ for $a \leq \hat{a}$ and $n_a = S_u(K)/w$ for $a > \hat{a}$.*

Proof. The working capital constraint $wn_a \leq S_a$ binds at the optimum since the firm maximizes output. ■

Claims 5–8 establish the four features stated in Proposition 2: front-loaded long-term debt payments, back-loaded dividends, increasing short-term debt during the early life cycle, and employment that tracks short-term debt.

OA.2.1.2 Proof of Proposition 3

Let $V_0(K, T, \xi)$ denote firm value at age 0 as a function of scale K, maturity T, and enforcement quality ξ. For each T, let $K_T(\xi)$ denote the maximum feasible scale. The optimal maturity $T^*(\xi)$ satisfies

$$\frac{V_0(K_{T^*+1}(\xi), T^* + 1, \xi)}{V_0(K_{T^*}(\xi), T^*, \xi)} < 1 \text{ and } \frac{V_0(K_{T^*}(\xi), T^*, \xi)}{V_0(K_{T^*-1}(\xi), T^* - 1, \xi)} \geq 1. \tag{OA.1}$$

$T^*(\xi)$ *is non-decreasing in* ξ. Higher enforcement relaxes financing constraints, so $K_T(\xi') \geq K_T(\xi)$ for $\xi' > \xi$ and any T, and V_0 is increasing in both K and ξ. Suppose for contradiction that $T^*(\xi') < T^*(\xi)$ for some $\xi' > \xi$. Then $T^*(\xi') + 1 \leq T^*(\xi)$, and the optimality condition at ξ gives

$$\frac{V_0(K_{T^*(\xi')+1}(\xi), T^*(\xi') + 1, \xi)}{V_0(K_{T^*(\xi')}(\xi), T^*(\xi'), \xi)} \geq 1. \tag{OA.2}$$

Since $K_T(\xi') \geq K_T(\xi)$ and V_0 is increasing in ξ:

$$\frac{V_0(K_{T^*(\xi')+1}(\xi'), T^*(\xi') + 1, \xi')}{V_0(K_{T^*(\xi')}(\xi'), T^*(\xi'), \xi')} \geq \frac{V_0(K_{T^*(\xi')+1}(\xi), T^*(\xi') + 1, \xi)}{V_0(K_{T^*(\xi')}(\xi), T^*(\xi'), \xi)} \geq 1, \tag{OA.3}$$

contradicting the optimality of $T^*(\xi')$ at ξ'.

$\Delta T^* \in \{0, 1\}$ *for any change* $\Delta\xi$. Since T^* takes integer values and is non-decreasing, $\Delta T \geq 0$. Suppose $\Delta T \geq 2$ for an increase from ξ to $\xi + \Delta\xi$. Then the maturity $T^*(\xi) + 1$ is skipped: it is suboptimal at ξ but dominated at $\xi + \Delta\xi$ by $T^*(\xi) + 2$ or higher. By the implicit function theorem, $K_T(\xi)$ is continuously differentiable in ξ (the binding enforcement constraint defines K_T implicitly through a smooth equation with nonzero partial derivative $\partial V_0/\partial K > 0$). The value ratio $V_0(K_{T+1}(\xi), T + 1, \xi)/V_0(K_T(\xi), T, \xi)$ is therefore continuous in ξ. At ξ, this ratio is strictly less than 1 for $T = T^*(\xi)$; by continuity, it remains below 1 in a neighborhood of ξ. A jump of 2 or more would require the ratio to cross 1 at $T^*(\xi) + 1$, which contradicts continuity for small $\Delta\xi$. For large $\Delta\xi$, the argument applies iteratively over subintervals.

OA.2.1.3 Proof of Proposition 4

Proof. Define the set of feasible discrete scales and maturities:

$$F = \{(T, K_T) \mid K_T \text{ is the maximum scale under full repayment } (\gamma = 1) \text{ at maturity } T\}. \tag{OA.4}$$

For each $(T, K_T) \in F$, firm value under full repayment is

$$V(T) = \frac{(\beta\rho)^{T+1}}{1-\beta\rho} \cdot [1-\eta(1-\alpha)] \cdot K_T^{\frac{\eta\alpha}{1-\eta(1-\alpha)}} z^{\frac{1}{1-\eta(1-\alpha)}} \left(\frac{\eta(1-\alpha)}{(1+r_f)w}\right)^{\frac{\eta(1-\alpha)}{1-\eta(1-\alpha)}}. \tag{OA.5}$$

For small T, $V(T)$ increases with the rising feasible scale K_T. As T grows, discounting dominates and $V(T)$ declines. There exists a unique

$$\tilde{T} = \min\left\{T \in \mathbb{N} \mid \frac{V(T+1)}{V(T)} < 1\right\}. \tag{OA.6}$$

Let $\hat{a}_0$ denote the ELC duration under full repayment at maturity $\tilde{T}$:

$$\hat{a}_0 = \left\lceil \tilde{T} + 1 - \frac{\log\left(\frac{(1-\beta\rho)(1-\xi)}{(1-\eta(1-\alpha))\beta\rho}\right)}{\log(\beta\rho)} \right\rceil. \tag{OA.7}$$

For scales $K \in [K_{\tilde{T}}, K_{\tilde{T}+1}]$, there exists a unique $\gamma(K) \in [0,1]$ satisfying (PCF):

$$(1+r_f)p_k K = \sum_{a=0}^{\tilde{T}} (\beta\rho)^a \pi_a(K) + (\beta\rho)^{\tilde{T}+1}\gamma(K)\pi_u(K). \tag{OA.8}$$

The function $\gamma(K)$ is continuous and strictly increasing, with $\gamma(K_{\tilde{T}}) = 0$ and $\gamma(K_{\tilde{T}+1}) = 1$.

The ELC duration under maturity $\tilde{T}+1$ with partial repayment γ is

$$\hat{a}(\gamma) = \left\lceil \tilde{T} + 1 - \frac{\log\left(\frac{(1-\beta\rho)(1-\xi)}{(1-\eta(1-\alpha))[\beta\rho+(1-\beta\rho)(1-\gamma)]}\right)}{\log(\beta\rho)} \right\rceil. \tag{OA.9}$$

As γ decreases from 1 to 0, the term $(1-\gamma)(1-\beta\rho) + \beta\rho$ increases from $\beta\rho$ to 1, which can shift the ceiling function by at most 1. Hence $\hat{a}(\gamma) \in \{\hat{a}_0, \hat{a}_0 + 1\}$.

Let $\bar{\gamma}$ be the threshold where $\hat{a}(\gamma)$ transitions from $\hat{a}_0 + 1$ to $\hat{a}_0$:

$$\bar{\gamma} = 1 - \frac{(\beta\rho)^{\tilde{T}+1-\hat{a}_0}(1-\xi) - \beta\rho}{(1-\beta\rho)}. \tag{OA.10}$$

By monotonicity of $\gamma(K)$, there exists a unique threshold $\bar{K} \in [K_{\tilde{T}}, K_{\tilde{T}+1}]$ with $\gamma(\bar{K}) = \bar{\gamma}$.

Firm value is

$$V(K,\gamma) = \frac{(\beta\rho)^{\tilde{T}+1}\pi_u(K)}{1-\beta\rho}[1-(1-\beta\rho)\gamma]. \tag{OA.11}$$

The first-order condition for optimal scale balances marginal benefit against the cost of delayed dividends:

$$[1-(1-\beta\rho)\gamma(K)]\frac{\partial \pi_u(K)}{\partial K} = (1-\beta\rho)\gamma'(K)\pi_u(K). \tag{OA.12}$$

The left side is the net marginal benefit of increased scale (gross profit gain less the fraction diverted to debt service). The right side is the marginal cost from a higher repayment fraction. Existence of a solution follows from continuity of $V(K,\gamma(K))$ on $[K_{\tilde{T}}, K_{\tilde{T}+1}]$.

Let K^* maximize $V(K,\gamma(K))$. The optimal maturity is $T^* = \tilde{T} + \mathbf{1}_{\gamma(K^*)>0}$, the optimal repayment share is $\gamma^* = \gamma(K^*)$, and the ELC duration is $\hat{a}^* = \hat{a}_0 + \mathbf{1}_{\gamma^*>\bar{\gamma}}$. ■

OA.2.1.4 Proof of Proposition 5

Part 1: Maximum feasible scale is increasing in enforcement.

For any fixed T, the maximum feasible scale K_T is determined by the binding enforcement constraint at $a = 0$:

$$(1-\xi)z\left[K_T^{\alpha}\left(\frac{S_0}{w}\right)^{1-\alpha}\right]^{\eta} = V_0(K_T, T). \tag{OA.13}$$

Taking the total derivative with respect to ξ and solving yields $\partial K_T/\partial\xi > 0$, since the numerator is positive (higher ξ directly relaxes the constraint and $\partial S_0/\partial\xi > 0$ further relaxes it) and the denominator is positive ($\partial V_0/\partial K > 0$ exceeds the marginal tightening of the constraint from higher output).

Part 2: Optimal maturity is weakly increasing in enforcement.

This follows from Proposition 3, already established above. The true optimal maturity $T^*(\xi) = \tilde{T}(\xi) + 1$ with partial repayment, and $\tilde{T}(\xi)$ is non-decreasing in ξ, so $T^*(\xi)$ is also non-decreasing with $\Delta T^*(\xi) \in \{0, 1\}$.

Part 3: Scale and value effects offset in ELC duration.

The ELC duration formula

$$\hat{a} = \left\lceil T^* + 1 - \frac{\ln\left(\frac{(1-\beta\rho)}{(1-\xi)(1-\beta\rho)}\right)}{\ln(\beta\rho)} \right\rceil \tag{OA.14}$$

does not contain K explicitly. Both the left side (output) and right side (continuation value) of the enforcement constraint scale with $K^{\eta\alpha/(1-\eta(1-\alpha))}$, so the direct effects of scale on ELC duration cancel. The primary channel through which scale affects ELC duration is the maturity T^*.

OA.2.1.5 Proof of Proposition 6

We prove that $\Delta\ell_a^S \equiv \ell_{a+1}^S - \ell_a^S > 0$ and $\Delta\ell_a^L \equiv \ell_{a+1}^L - \ell_a^L < 0$ for $a < \hat{a}(\xi)$ by contradiction.

Recall the definitions $\ell_a^S = S_a/(V_a + S_a + L_a)$ and $\ell_a^L = L_a/(V_a + S_a + L_a)$.

Short-term leverage. Suppose $\ell_{a+1}^S \leq \ell_a^S$ for some $a < \hat{a}$. Cross-multiplying:

$$S_{a+1}(V_a + S_a + L_a) \leq S_a(V_{a+1} + S_{a+1} + L_{a+1}).$$

Expanding and canceling $S_{a+1}S_a$:

$$S_{a+1}V_a - S_aV_{a+1} \leq S_aL_{a+1} - S_{a+1}L_a.$$

From Proposition 2, during the ELC: $S_{a+1} > S_a$, $V_{a+1} > V_a$, and $L_{a+1} < L_a$. Since $L_{a+1}/L_a < 1 < S_{a+1}/S_a$, the right side satisfies $S_aL_{a+1} - S_{a+1}L_a < 0$. Hence the left side must be negative: $S_{a+1}/S_a < V_{a+1}/V_a$.

We now compute growth rates. From Proposition 2:

$$S_a = \left[\frac{(\beta\rho)^{T(K)+1-a}}{1-\beta\rho}\frac{1}{1-\xi}\right]^{\frac{1}{\eta(1-\alpha)}} S_u(K, z),$$
$$V_a = (\beta\rho)^{T+1-a}\frac{\pi_u(K, z)}{1-\beta\rho}.$$

These formulas are derived under $\gamma = 1$ for notational simplicity; the growth rates S_{a+1}/S_a and V_{a+1}/V_a are identical for any $\gamma \in [0, 1]$ because γ affects only the level of V_a through the factor $\beta\rho + (1-\beta\rho)(1-\gamma)$, which cancels in the ratio V_{a+1}/V_a. The growth rates are

$$\frac{S_{a+1}}{S_a} = \left(\frac{1}{\beta\rho}\right)^{\frac{1}{\eta(1-\alpha)}}, \qquad \frac{V_{a+1}}{V_a} = \frac{1}{\beta\rho}.$$

Since $\eta(1-\alpha) < 1$ and $\beta\rho < 1$, we have $(1/\beta\rho)^{1/\eta(1-\alpha)} > 1/\beta\rho$, so $S_{a+1}/S_a > V_{a+1}/V_a$. This contradicts $S_{a+1}/S_a < V_{a+1}/V_a$.

Long-term leverage. Suppose $\ell^L_{a+1} \geq \ell^L_a$ for some $a < \hat{a}$. Cross-multiplying:

$$L_{a+1}V_a - L_aV_{a+1} \geq L_aS_{a+1} - L_{a+1}S_a.$$

The left side satisfies $L_{a+1}V_a - L_aV_{a+1} = L_aV_a(L_{a+1}/L_a - V_{a+1}/V_a) < 0$ (since $L_{a+1}/L_a < 1 < V_{a+1}/V_a$). The right side satisfies $L_aS_{a+1} - L_{a+1}S_a = L_aS_a(S_{a+1}/S_a - L_{a+1}/L_a) > 0$. A negative quantity cannot exceed a positive quantity, yielding a contradiction.

OA.2.2 Proof of Proposition 8

At any age a during the ELC, total assets decompose as $A_a = V_a + S_a + L_a$ with $V_a = \Gamma_a\pi_u(K)$, $S_a = \Theta_a S_u(K,z)$, and $L_a = \Omega_a\pi_u(K)$, where Γ_a and Ω_a are present-value factors depending on contract structure $(T,\gamma,\hat{a})$. Since $\pi_u(K,z) = \kappa S_u(K,z)$ with $\kappa = (1+r_f)(1-\eta(1-\alpha))/(\eta(1-\alpha))$, scale and productivity cancel in balance sheet ratios.

Short-term leverage. Write $\ell^S_a = (V_a/S_a + 1 + L_a/S_a)^{-1}$. Using $V_a = \Gamma_a\kappa S_u$ and $S_a = \Theta_a S_u$, the ratio $V_a/S_a = \Gamma_a\kappa/\Theta_a$ decreases with enforcement because Θ_a rises and Γ_a weakly falls. Similarly, $L_a/S_a = \Omega_a\kappa/\Theta_a$ decreases because Θ_a rises faster than Ω_a during the ELC. Both ratios falling implies ℓ^S_a rises.

Long-term leverage. The decomposition $\ell^L_a = \lambda_a(1-\ell^S_a)$ with $\lambda_a \equiv L_a/(L_a+V_a)$ separates two effects. For the debt-to-capital ratio, write

$$L_a + V_a = (\Omega_a + \Gamma_a)\pi_u = (\Phi^{FB} - \mathcal{D}_a)\pi_u,$$

where $\Phi^{FB} \equiv (1-\beta\rho)^{-1}$ is the first-best present value factor and $\mathcal{D}_a \equiv \Phi^{FB} - \Omega_a - \Gamma_a > 0$ is the distortion wedge from constrained operation. Better enforcement shrinks $\mathcal{D}_a$ toward zero. The equity factor $\Gamma_a = (\beta\rho)^{T-a}(1-\gamma)/(1-\beta\rho)$ depends on enforcement only through (T^*,γ^*) and is relatively stable, so the reduction in $\mathcal{D}_a$ accrues primarily to Ω_a, and $\lambda_a = \Omega_a/(\Omega_a+\Gamma_a)$ increases with ξ.

The scaling factor $(1 - \ell_a^S)$ lies in $[0.85, 1]$ for empirically relevant calibrations with $\ell_a^S \leq 0.15$. Differentiating:

$$\frac{\partial \ell_a^L}{\partial \xi} = \frac{\partial \lambda_a}{\partial \xi}(1 - \ell_a^S) - \lambda_a \frac{\partial \ell_a^S}{\partial \xi},$$

which is positive if and only if

$$\frac{1}{\lambda_a}\frac{\partial \lambda_a}{\partial \xi} > \frac{1}{1 - \ell_a^S}\frac{\partial \ell_a^S}{\partial \xi}.$$

The left side is strictly positive. The right side is bounded: discrete jumps in (T^*, γ^*) can induce discontinuities in ℓ_a^S, but these are bounded by ℓ_a^S itself. For ℓ_a^S sufficiently small, the condition holds. Under the maintained assumption that $\ell_a^S \leq 0.15$ (consistent with the baseline calibration where short-term leverage is approximately 10% of total assets), the first term dominates and $\partial \ell_a^L / \partial \xi > 0$.

OA.3 Stochastic Model: Formal Development

This appendix provides formal proofs for the stochastic extension of Section 4 and derives closed-form expressions for every entry-age balance-sheet object. Because the enforcement constraint binds at every history during the constrained phase, the deterministic contract structure of Section 3 applies path by path. Stochastic productivity places a distribution over the terminal contract objects (maturity T and repayment share γ) rather than altering their functional form. We work with the recursive formulation of Albuquerque and Hopenhayn (2004).

The primitive parameters are η, α, β, ρ, w, p_k, r_f, and ξ. Throughout, K is chosen once at entry and remains fixed. The firm-level state variables at entry are the permanent type χ and the entry shock $\tilde{z}_0$. No composite parameter or shorthand beyond $\theta \equiv 1/(1-\eta(1-\alpha))$ is introduced; all expressions are written in terms of primitive parameters. The deterministic model of Section 3 is the special case $\tilde{z}_a = 1$ a.s.

OA.3.1 Environment

The technology, timing, and outside option are as in Section 3. We restate the primitives and introduce the probability structure.

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. A firm draws a permanent type $\chi \in \mathbb{R}_+$ at entry from a distribution F with $\mathbb{E}[\chi^\theta] < \infty$. Transitory shocks $\{\tilde{z}_a\}_{a \geq 0}$ are i.i.d. with distribution G, independent of χ. Normalize $\mathbb{E}[\tilde{z}] = 1$. Total productivity at age a is $z_{ia} = \chi \tilde{z}_{ia}$. The history up to age a is $\tilde{z}^a \equiv (\tilde{z}_0, \ldots, \tilde{z}_a)$, and $\mathcal{F}_a \equiv \sigma(\chi, \tilde{z}^a)$ the information set at age a. All contract objects at age a are $\mathcal{F}_a$-measurable. Write $\mathbb{E}_a[\,\cdot\,] \equiv \mathbb{E}[\,\cdot \mid \mathcal{F}_a]$. Survival is i.i.d. with probability $\rho \in (0,1)$ each period.

Recall the productivity elasticity

$$\theta \equiv \frac{1}{1-\eta(1-\alpha)} > 1. \tag{OA.15}$$

Define the *productivity moment* used throughout:

$$\mu_\theta \equiv \mathbb{E}[\tilde{z}^\theta] < \infty. \tag{OA.16}$$

In the deterministic special case $\tilde{z}_a = 1$ a.s., $\mu_\theta = 1$.

Assumption 4 (Bounded support) *The distribution G has support contained in $[\underline{z}, \bar{z}]$ for some $0 < \underline{z} < \bar{z} < \infty$.*

Technology and optimal labor demand. The production function is

$$y_a \;=\; \chi\,\tilde{z}_{ia}\left[K^{\alpha}\left(\frac{S_a}{w}\right)^{1-\alpha}\right]^{\eta}. \tag{OA.17}$$

Labor is financed entirely by short-term borrowing: $wn_a = S_a$. For given K and productivity $\chi\tilde{z}_a$, the firm maximizes $y_a - (1+r_f)S_a$ with respect to S_a. The first-order condition is

$$\eta(1-\alpha)\,\frac{y_a}{S_a} \;=\; (1+r_f), \tag{OA.18}$$

so the revenue product of the short-term loan equals its gross cost. Solving explicitly:

$$S_u(K,\chi,\tilde{z}) \;=\; \left(\frac{\eta(1-\alpha)}{1+r_f}\right)^{\eta(1-\alpha)\theta} w^{-\eta(1-\alpha)\theta}\,(\chi\tilde{z})^{\theta}\,K^{\eta\alpha\theta}. \tag{OA.19}$$

A consequence of (OA.18) used repeatedly below is

$$(1+r_f)\,S_u(K,\chi,\tilde{z}) \;=\; \eta(1-\alpha)\,y_u(K,\chi,\tilde{z}), \tag{OA.20}$$

where y_u denotes output evaluated at S_u.

Unconstrained output and profits. Substituting S_u into (OA.17):

$$y_u(K,\chi,\tilde{z}) \;=\; \left(\frac{\eta(1-\alpha)}{(1+r_f)w}\right)^{\eta(1-\alpha)\theta}(\chi\tilde{z})^{\theta}\,K^{\eta\alpha\theta}. \tag{OA.21}$$

Profits net of the working-capital interest cost are, using (OA.20):

$$\pi_u(K,\chi,\tilde{z}) \;=\; y_u - (1+r_f)S_u \;=\; \big(1-\eta(1-\alpha)\big)\,y_u(K,\chi,\tilde{z}). \tag{OA.22}$$

Substituting (OA.21):

$$\pi_u(K,\chi,\tilde{z}) \;=\; \pi_u(1,1,1)\cdot(\chi\tilde{z})^{\theta}\,K^{\eta\alpha\theta}, \tag{OA.23}$$

where the *profit scale* evaluated at unit talent and unit productivity collects all price parameters:

$$\pi_u(1,1,1) \;\equiv\; \big(1-\eta(1-\alpha)\big)\left(\frac{\eta(1-\alpha)}{(1+r_f)w}\right)^{\eta(1-\alpha)\theta}. \tag{OA.24}$$

Expected profits under i.i.d. shocks satisfy

$$\mathbb{E}\big[\pi_u(K,\chi,\tilde{z})\big] \;=\; \mu\cdot\pi_u(1,1,1)\cdot\chi^{\theta}\,K^{\eta\alpha\theta}, \tag{OA.25}$$

and the present value of expected profits under permanent operation is

$$\Pi_u(K,\chi) \equiv \frac{\mathbb{E}[\pi_u(K,\chi,\tilde{z})]}{1-\beta\rho} = \frac{\mu\cdot\pi_u(1,1,1)}{1-\beta\rho}\,\chi^{\theta}K^{\eta\alpha\theta}. \tag{OA.26}$$

OA.3.2 Contracting problem

OA.3.2.1 Sequential formulation

The contracting problem extends Section 3 to the stochastic setting: a contract specifies $S_a(\tilde{z}^a) \geq 0$ and a transfer $q_a(\tilde{z}^a) \geq 0$ for each history. The continuation values are

$$V_a(\tilde{z}^a) \equiv \mathbb{E}\left[\sum_{j\geq a}(\beta\rho)^{j-a}\big(\pi_j(\tilde{z}^j)-q_j(\tilde{z}^j)\big)\,\middle|\,\mathcal{F}_a\right], \qquad B_a(\tilde{z}^a) \equiv \mathbb{E}\left[\sum_{j\geq a}(\beta\rho)^{j-a}\,q_j(\tilde{z}^j)\,\middle|\,\mathcal{F}_a\right]. \tag{OA.27}$$

The optimal contract maximizes V_0 subject to

$$\text{(PCF)}\quad B_0 \geq p_k(1+r_f)K, \tag{OA.28}$$

$$\text{(NNC)}\quad 0 \leq q_a(\tilde{z}^a) \leq \pi_a(\tilde{z}^a) \quad \text{for all } a, \tilde{z}^a, \tag{OA.29}$$

$$\text{(LE)}\quad V_a(\tilde{z}^a) \geq (1-\xi)\,y_a(\tilde{z}^a) \quad \text{for all } a, \tilde{z}^a, \tag{OA.30}$$

$$\text{(WCC)}\quad S_a(\tilde{z}^a) \geq 0 \quad \text{for all } a, \tilde{z}^a. \tag{OA.31}$$

OA.3.2.2 Recursive formulation

We use the lender's remaining claim as the state variable:

$$L_a \equiv B_a(\tilde{z}^a) = \mathbb{E}\left[\sum_{j\geq a}(\beta\rho)^{j-a}\,q_j\,\middle|\,\mathcal{F}_a\right], \tag{OA.32}$$

satisfying $L_a = q_a + \beta\rho\,\mathbb{E}[L_{a+1}\mid\mathcal{F}_a]$. At entry, $L_0 = p_k(1+r_f)K$.

Homogeneity of flow objects.

Lemma 1 (Flow homogeneity) *The quantities S_u, y_u, π_u, and $(1-\xi)\,y$ are each proportional to $\chi^\theta K^{\eta\alpha\theta}$ for given $(\tilde{z}, w)$.*

Proof. From (OA.19), $S_u \propto \chi^\theta K^{\eta\alpha\theta}$. Since $\pi_u = \pi_u(1,1,1)\,\chi^\theta \tilde{z}^\theta K^{\eta\alpha\theta}$ and $y_u = \pi_u/(1-\eta(1-\alpha))$, output inherits the same scaling. The outside option $(1-\xi)y$ inherits it for any feasible $S \le S_u$. ■

Normalized Bellman equation. Dividing by $\chi^\theta K^{\eta\alpha\theta}$ removes (χ, K) from all flow objects. Define $\ell \equiv L/(\chi^\theta K^{\eta\alpha\theta})$ and $v(\ell, \tilde{z}) \equiv V(L, \chi, \tilde{z})/(\chi^\theta K^{\eta\alpha\theta})$. A continuation contract chooses $\vartheta \in [0,1]$, $\hat{q} \ge 0$, and $\tilde{z}' \mapsto \ell'(\tilde{z}') \ge 0$. Define

$$\hat{\pi}(\vartheta, \tilde{z}) \equiv \pi_u(1,1,1)\,\tilde{z}^\theta\,\frac{\vartheta^{\eta(1-\alpha)} - \eta(1-\alpha)\,\vartheta}{1-\eta(1-\alpha)}. \tag{OA.33}$$

Definition 1 (Bellman equation) *For fixed (K, ξ),*

$$v(\ell, \tilde{z}) = \sup_{\vartheta\in[0,1],\,\hat{q},\,\ell'(\cdot)} \left\{\hat{d} + \beta\rho\,\mathbb{E}[v(\ell'(\tilde{z}'), \tilde{z}')]\right\} \tag{OA.34}$$

subject to

$$\hat{d} = \hat{\pi}(\vartheta, \tilde{z}) - \hat{q} \ge 0, \tag{OA.35}$$

$$\hat{q} \ge 0, \tag{OA.36}$$

$$\ell = \hat{q} + \beta\rho\,\mathbb{E}[\ell'(\tilde{z}')], \tag{OA.37}$$

$$\hat{d} + \beta\rho\,\mathbb{E}[v(\ell'(\tilde{z}'), \tilde{z}')] \ge \frac{(1-\xi)}{1-\eta(1-\alpha)}\,\pi_u(1,1,1)\,\tilde{z}^\theta\,\vartheta^{\eta(1-\alpha)}, \tag{OA.38}$$

$$\ell'(\tilde{z}') \ge 0 \quad \textit{for all } \tilde{z}'. \tag{OA.39}$$

OA.3.2.3 Existence and uniqueness

Lemma 2 (Bounded state space) *Under any feasible contract, $\ell \in [0, \bar{\ell}]$ where $\bar{\ell} \equiv \mu \cdot \pi_u(1,1,1)/(1-\beta\rho)$.*

Proof. By (OA.29), $q_a \le \pi_u$ for all a. Hence $L_0 \le \mu \cdot \pi_u(1,1,1)\,\chi^\theta K^{\eta\alpha\theta}/(1-\beta\rho)$. Dividing by $\chi^\theta K^{\eta\alpha\theta}$ gives $\ell_0 \le \bar{\ell}$, and the same bound applies at any age. ■

Proposition OA.1 (Existence and uniqueness) *Under Assumption 4, the Bellman operator is a contraction on bounded measurable functions on* $[0, \bar{\ell}] \times [\underline{z}, \bar{z}]$ *with the sup norm. The unique fixed point* v *is weakly decreasing in* ℓ *and weakly increasing in* $\tilde{z}$.

Proof. *Contraction.* Blackwell's sufficient conditions hold: monotonicity is immediate, and discounting follows from $\mathcal{T}(v + c) = \mathcal{T}v + \beta\rho\, c$. Hence the operator contracts with modulus $\beta\rho < 1$.

Monotonicity in ℓ. A lower ℓ relaxes the promise-keeping constraint, so $v(\ell_1, \tilde{z}) \geq v(\ell_2, \tilde{z})$ for $\ell_2 > \ell_1$.

Monotonicity in $\tilde{z}$. Higher $\tilde{z}$ increases normalized profits faster than the outside option (the former captures the full return, the latter only the $(1 - \xi)$ share), so any feasible allocation at $\tilde{z}_1$ remains feasible at $\tilde{z}_2 > \tilde{z}_1$ with weakly higher dividends.

Existence of an optimal measurable policy follows from standard results for discounted dynamic programs on Borel state spaces (see, e.g., Stokey et al., 1989, Chapter 9). ■

Proposition OA.2 (Solution homogeneity) *The normalized Bellman equation depends on* $(\ell, \tilde{z}, \xi, \beta\rho)$ *and is free of* (χ, K). *The policy rules* $\Theta(\ell, \tilde{z})$, $(\hat{a}, T, \gamma)$ *depend on* (χ, K) *only through* ℓ. *Consequently,* $(\pi_a, S_a, V_a) \propto \chi^{\theta} K^{\eta\alpha\theta}$ *for any history.*

Proof. The operator maps (χ, K)-free functions to (χ, K)-free functions by Lemma 1. The unique fixed point and associated policies therefore depend on (χ, K) only through ℓ. Since $S_a = \Theta_a S_u$ and $\pi_a = g(\Theta_a)\pi_u$ with $S_u, \pi_u \propto \chi^{\theta} K^{\eta\alpha\theta}$, the allocation inherits the same scaling. ■

OA.3.3 Structure of the optimal allocation

OA.3.3.1 Scale distortion and profit efficiency

Define $\Theta \in (0, 1]$ by $S = \Theta \cdot S_u(K, \chi, \tilde{z})$.

Lemma 3 (Scaling of output and profits) *For any* $\Theta \in (0, 1]$,

$$y = \Theta^{\eta(1-\alpha)}\, y_u(K, \chi, \tilde{z}), \qquad \pi = g(\Theta)\, \pi_u(K, \chi, \tilde{z}), \tag{OA.40}$$

where the profit efficiency function *is*

$$g(\Theta) \equiv \frac{\Theta^{\eta(1-\alpha)} - \eta(1-\alpha)\,\Theta}{1 - \eta(1-\alpha)} \;\in\; (0, 1) \quad \textit{for } \Theta \in (0, 1), \qquad g(1) = 1. \tag{OA.41}$$

The function g satisfies $g(0) = 0$, $g' > 0$ on $(0, 1)$, and g is strictly concave.

Proof. Setting $S = \Theta S_u$: $y = \Theta^{\eta(1-\alpha)} y_u$. For profits: $\pi = \Theta^{\eta(1-\alpha)} y_u - \Theta(1 + r_f)S_u = (\Theta^{\eta(1-\alpha)} - \eta(1-\alpha)\,\Theta)\, y_u = g(\Theta)\,\pi_u$, using $(1 + r_f)S_u = \eta(1-\alpha)\, y_u$. The properties follow from $g'(\Theta) = \eta(1-\alpha)\,(\Theta^{\eta(1-\alpha)-1} - 1)/(1 - \eta(1-\alpha)) > 0$ for $\Theta < 1$ and $g''(\Theta) = \eta(1-\alpha)\,(\eta(1-\alpha) - 1)\,\Theta^{\eta(1-\alpha)-2}/(1 - \eta(1-\alpha)) < 0$. ■

OA.3.3.2 Characterization of the optimal contract

Proposition OA.3 (Enforcement binds when $\Theta < 1$) *If $\xi \in (0, 1)$ and $\Theta < 1$ at state $(\ell, \tilde{z})$, then (OA.38) holds with equality.*

Proof. If (OA.38) is slack and $\Theta < 1$, increasing Θ by ε raises profits while maintaining feasibility for small ε, contradicting optimality. ■

Proposition OA.4 (No dividends while enforcement binds) *If (OA.38) binds, there exists an optimal allocation with $\hat{d} = 0$.*

Proof. Fix an optimal allocation with $\hat{d}^* > 0$ and binding enforcement. Increase $\hat{q}$ by ε and reduce $\ell'(\cdot)$ uniformly by $\varepsilon/(\beta\rho)$ to preserve (OA.37). For enforcement: the change in the left side of (OA.38) is $-\varepsilon + \beta\rho\,\mathbb{E}[v(\ell'^* - \varepsilon/(\beta\rho), \tilde{z}') - v(\ell'^*, \tilde{z}')]$. At state $(\ell' - \varepsilon/(\beta\rho), \tilde{z}')$, replicating the allocation at $(\ell', \tilde{z}')$ and distributing $\varepsilon/(\beta\rho)$ as dividends is feasible, so $v(\ell' - \varepsilon/(\beta\rho), \tilde{z}') \geq v(\ell', \tilde{z}') + \varepsilon/(\beta\rho)$. The net change is at least 0. Iterating until $\hat{d} = 0$ is weakly improving. ■

Proposition OA.5 (Front-loaded repayment) *At any state $(\ell, \tilde{z})$, there exists an optimal allocation with $\hat{q} = \min\{\hat{\pi}(\Theta, \tilde{z}),\ \ell\}$. If $\ell \leq \hat{\pi}(1, \tilde{z})$ and enforcement is slack at $\Theta = 1$, then $\Theta = 1$, $\hat{q} = \ell$, and $\ell'(\cdot) \equiv 0$.*

Proof. When enforcement binds, Propositions OA.3–OA.4 give $\hat{q} = \hat{\pi}$ and $\hat{d} = 0$. When enforcement is slack and $\hat{q} < \min\{\hat{\pi}, \ell\}$, the same perturbation (increase $\hat{q}$, reduce ℓ') is weakly improving by the argument in Proposition OA.4. When $\ell \leq \hat{\pi}(1, \tilde{z})$ and enforcement is slack at $\Theta = 1$, setting $\Theta = 1$, $\hat{q} = \ell$, $\ell' \equiv 0$ is feasible and achieves the unconstrained value. ■

OA.3.3.3 Three-phase structure and realization map

Each realized productivity history $\tilde{z}^\infty$ maps to a triple

$$\tilde{z}^\infty \longmapsto \big(T(\tilde{z}^\infty),\ \gamma(\tilde{z}^\infty),\ \hat{a}(\tilde{z}^\infty)\big), \tag{OA.42}$$

where T, γ, and $\hat{a}$ are defined as follows.

Definition 2 (Maturity and terminal repayment) $T(\tilde{z}^\infty) \equiv \inf\{a \geq 0 : L_{a+1} = 0\}$. *At maturity, $q_T = L_T$ and $L_{T+1} = 0$. The terminal repayment share is $\gamma(\tilde{z}^\infty) \equiv q_T/\pi_u(K, \chi, \tilde{z}_T) \in (0, 1]$.*

Definition 3 (ELC exit age) $\hat{a}(\tilde{z}^\infty) \equiv \inf\{a \geq 0 : \Theta_a = 1\}$.

A firm with high realized productivity retires debt faster and reaches maturity sooner. For $a < T$, all profits are transferred ($q_a = \pi_a$); at maturity, $q_T = \gamma\pi_u$; for $a > T$, no payments are made. The ELC exit age $\hat{a}$ is the first age at which enforcement ceases to bind; once the firm exits the constrained regime, it remains unconstrained.

Proposition OA.6 (Three-phase structure) *Under the optimal allocation, $\hat{a} \leq T$, and:*

(i) Constrained phase ($a < \hat{a}$): Enforcement binds, $\Theta_a < 1$, $d_a = 0$, $q_a = g(\Theta_a)\pi_u$.

(ii) Unconstrained scale with outstanding debt ($\hat{a} \leq a < T$): $\Theta_a = 1$, $S_a = S_u$, $L_a > 0$, $q_a = \min\{\pi_u, L_a\}$.

(iii) Fully unconstrained ($a \geq T$): $L_a = 0$, $q_a = 0$, $S_a = S_u$, $d_a = \pi_u$.

Proof. Phase (i) follows from Propositions OA.3–OA.4. Phase (ii): at $\hat{a}$, $\Theta_{\hat{a}} = 1$ but $L_{\hat{a}} > 0$ in general; front-loading (Proposition OA.5) sets $q_a = \min\{\pi_u, L_a\}$. Phase (iii): once $L_a = 0$, $\hat{q} = 0$ and $\ell' = 0$. That $\hat{a} \leq T$: at maturity, L_T is retired in full, so $\Theta_T = 1$, hence $\hat{a} \leq T$. ■

OA.3.3.4 Implementation

Proposition OA.7 (Unique implementation) *Under Assumptions 1–3, the optimal allocation is uniquely implemented by a long-term debt contract with face value $L_0 = p_k(1 + r_f)K$ and a sequence of short-term loans $\{S_a(\tilde{z}^a)\}_{a\geq 0}$.*

Proof. Follows from Proposition 1, which depends only on the entry-period payoff structure and is identical under stochastic and deterministic productivity. ■

Remark 1 (Post-$\hat{a}$ balance sheet convention) *In Phase (i), binding enforcement and the no-dividend result pin down both allocation and implementation uniquely. In Phase (ii), the allocation is determinate but implementation admits alternatives; we adopt* $q_a = \min\{\pi_u, L_a\}$*. In Phase (iii), no further choices arise.*

OA.3.4 Value decomposition and constraint multiplier

Proposition OA.8 (Value decomposition) *For any* $a \leq T$*,*

$$V_a(\tilde{z}^a) = \mathbb{E}\left[(\beta\rho)^{T-a}(1-\gamma)\pi_u(K,\chi,\tilde{z}_T) + \sum_{j>T}(\beta\rho)^{j-a}\pi_u(K,\chi,\tilde{z}_j) \,\middle|\, \mathcal{F}_a\right]. \tag{OA.43}$$

Proof. By Proposition OA.6: $d_j = 0$ for $j < T$; $d_T = (1-\gamma)\pi_u$; $d_j = \pi_u$ for $j > T$. Substituting into $V_a = \mathbb{E}[\sum_{j\geq a}(\beta\rho)^{j-a}d_j \mid \mathcal{F}_a]$ yields (OA.43). ■

Because $\pi_u(K,\chi,\tilde{z}) \propto \chi^\theta K^{\eta\alpha\theta}$ by (OA.23), the factor $\pi_u(1,1,1)\cdot\chi^\theta K^{\eta\alpha\theta}$ can be pulled outside the expectation. Define the *stochastic maturity factor*:

$$M_a(\tilde{z}^a) \;\equiv\; (1-\beta\rho)\,\mathbb{E}_a\left[(\beta\rho)^{T(\tilde{z}^\infty)-a}\left((1-\gamma(\tilde{z}^\infty))\,\tilde{z}^\theta_{T(\tilde{z}^\infty)} + \frac{\beta\rho\,\mu}{1-\beta\rho}\right)\right]. \tag{OA.44}$$

In the deterministic case $\tilde{z}_a = 1$ a.s. with fixed (T,γ), equation (OA.44) reduces to $M_a^{\text{det}} = (\beta\rho)^{T-a}[\beta\rho + (1-\beta\rho)(1-\gamma)]$, the maturity factor of the baseline model.

Corollary 2 (Compact representation) *For any* $a \leq T$*,*

$$V_a(\tilde{z}^a) = \frac{M_a(\tilde{z}^a)}{1-\beta\rho}\cdot\pi_u(1,1,1)\cdot\chi^\theta\,K^{\eta\alpha\theta}. \tag{OA.45}$$

Proof. From (OA.43) and $\pi_u = \pi_u(1,1,1)\,\chi^\theta\tilde{z}^\theta K^{\eta\alpha\theta}$, condition on $\mathcal{F}_T$ for the post-T sum: $\mathbb{E}[\tilde{z}^\theta_j] = \mu$ for $j > T$ by i.i.d. shocks. Collecting terms and factoring $\mu/(1-\beta\rho)$ yields (OA.45)–(OA.44). ■

Proposition OA.9 (Constraint multiplier) *During the constrained phase* $(a < \hat{a})$*,*

$$\Theta_a(\tilde{z}^a) = \left[\frac{(1-\eta(1-\alpha))\,\mu\,M_a(\tilde{z}^a)}{(1-\xi)(1-\beta\rho)\,\tilde{z}^\theta_a}\right]^{1/\eta(1-\alpha)}. \tag{OA.46}$$

Proof. Binding enforcement gives $(1-\xi)\, y_a = V_a$. Using $y_a = \Theta_a^{\eta(1-\alpha)}\, \pi_u(1,1,1)\, (\chi \tilde{z}_a)^\theta K^{\eta\alpha\theta}/(1-\eta(1-\alpha))$ and $V_a = \pi_u(1,1,1)\, \chi^\theta K^{\eta\alpha\theta}\, M_a/(1-\beta\rho)$:

$$(1-\xi)\, \Theta_a^{\eta(1-\alpha)}\, \frac{\pi_u(1,1,1)\, \chi^\theta\, \tilde{z}_a^\theta\, K^{\eta\alpha\theta}}{1-\eta(1-\alpha)} = \frac{\mu \cdot \pi_u(1,1,1)\, \chi^\theta\, K^{\eta\alpha\theta}}{1-\beta\rho} \cdot M_a.$$

The terms $\pi_u(1,1,1)\, \chi^\theta K^{\eta\alpha\theta}$ cancel. Solving for $\Theta_a^{\eta(1-\alpha)}$ and raising to the power $1/\eta(1-\alpha)$ gives (OA.46). ∎

No permanent characteristics χ or scale K survive in (OA.46): both cancel algebraically from the enforcement condition. In the deterministic case $\tilde{z}_a = 1$, $M_a = M_a^{\text{det}}$, and (OA.46) recovers the deterministic constraint multiplier of Proposition 2:

$$\Theta_a^{\text{det}} \;=\; \left[\frac{(1-\eta(1-\alpha))\,(\beta\rho)^{T-a}[\beta\rho + (1-\beta\rho)(1-\gamma)]}{(1-\beta\rho)(1-\xi)}\right]^{1/\eta(1-\alpha)}. \qquad \text{(OA.47)}$$

Remark 2 (Decomposition) *The constraint multiplier admits* $\Theta_a = \Theta_a^{det} \cdot [M_a/M^{det}]^{1/\eta(1-\alpha)} \cdot [\mu/\tilde{z}_a^\theta]^{1/\eta(1-\alpha)}$. *The first adjustment captures path dependence (different histories lead to different distance to maturity); the second captures the contemporaneous shock (above-average productivity tightens the constraint).*

Lemma 4 (ELC exit condition) *The firm exits the ELC at the first age* $\hat{a}$ *such that* $(1-\eta(1-\alpha))\, \mu\, M_{\hat{a}}/[(1-\xi)(1-\beta\rho)\, \tilde{z}_{\hat{a}}^\theta] \geq 1$.

Proof. For $a < \hat{a}$, enforcement binds and the LHS is strictly less than 1. At $\hat{a}$ it reaches or exceeds 1, so $\Theta_{\hat{a}} = 1$. ∎

Constrained profits. Period profits during the constrained phase, using (OA.40) and (OA.20), are $\pi_a(\tilde{z}^a) = g(\Theta_a) \cdot \pi_u(K, \chi, \tilde{z}_a)$.

OA.3.4.1 Observable leverage

Corollary 3 (Leverage invariance under optimal entry scale) *Under optimal entry scale* $K^*(\chi, \xi) = \chi^{\theta/(1-\eta\alpha\theta)} \bar{K}(\xi)$*, leverage ratios* ℓ_a^S *and* ℓ_a^L *are independent of* χ.

Proof. Optimal entry scale satisfies $K^* \propto \chi^{\theta/(1-\eta\alpha\theta)}$ by homogeneity of the entry problem (Proposition OA.2). Hence $L_0 \propto K^* \propto \chi^{\theta/(1-\eta\alpha\theta)}$ and $\pi_u \propto \chi^\theta (K^*)^{\eta\alpha\theta} \propto \chi^{\theta/(1-\eta\alpha\theta)}$. All balance-sheet components scale identically, so ratios are χ-invariant. ∎

This invariance provides the exclusion restriction for cross-country identification. Under Proposition OA.7 and Remark 1, the observable balance sheet is

$$S_a = \Theta_a(\tilde{z}^a)\, S_u(K,\chi,\tilde{z}_a), \tag{OA.48}$$

$$L_a = L_0 - \sum_{j=0}^{a-1}(\beta\rho)^{-j} q_j(\tilde{z}^j), \tag{OA.49}$$

$$A_a = S_a + L_a + V_a. \tag{OA.50}$$

Short-term leverage $\ell_a^S \equiv S_a/A_a$ and long-term leverage $\ell_a^L \equiv L_a/A_a$ are χ-invariant by Corollary 3.

OA.3.5 NPV decomposition, debt capacity, and entry scale

This section expresses every entry-age balance-sheet object in closed form, using two summary statistics of the optimal contract: a *debt capacity* factor Λ and an *equity factor* Γ. The derivation traces how permanent talent χ, factor prices (w, p_k, r_f), and enforcement quality ξ enter each expression.

OA.3.5.1 L_0 as the NPV of promised payments

Long-term debt at entry is the expected discounted sum of all transfers from firm to lender:

$$L_0 \;\equiv\; \mathbb{E}_0\left[\sum_{a=0}^{T(\tilde{z}^\infty)} (\beta\rho)^a\, q_a(\tilde{z}^a)\right]. \tag{OA.51}$$

Under the optimal front-loaded schedule: $q_a = \pi_a(\tilde{z}^a)$ for $a < \hat{a}(\tilde{z}^\infty)$ (constrained phase); $q_a = \pi_u(K,\chi,\tilde{z}_a)$ for $\hat{a}(\tilde{z}^\infty) \le a < T(\tilde{z}^\infty)$ (unconstrained phase before maturity); and $q_T = \gamma(\tilde{z}^\infty)\,\pi_u(K,\chi,\tilde{z}_T)$. Splitting (OA.51) across these three phases:

$$L_0 \;=\; \mathbb{E}_0\left[\sum_{a=0}^{\hat{a}-1}(\beta\rho)^a\pi_a(\tilde{z}^a) + \sum_{a=\hat{a}}^{T-1}(\beta\rho)^a\pi_u(K,\chi,\tilde{z}_a) + (\beta\rho)^T\gamma\,\pi_u(K,\chi,\tilde{z}_T)\right], \tag{OA.52}$$

where $\hat{a}$, T, γ are shorthand for $\hat{a}(\tilde{z}^\infty)$, $T(\tilde{z}^\infty)$, $\gamma(\tilde{z}^\infty)$.

Every term in (OA.52) is proportional to $\pi_u(1,1,1)\cdot(\chi\tilde{z}_a)^\theta\, K^{\eta\alpha\theta}$ by (OA.23). Using

$\pi_a = g(\Theta_a) \cdot \pi_u$ and factoring out $\pi_u(1,1,1) \cdot \chi^\theta K^{\eta\alpha\theta}$:

$$L_0 \;=\; \Lambda(\beta,\rho,\xi,G) \cdot \pi_u(1,1,1) \cdot \chi^\theta\, K^{\eta\alpha\theta}, \tag{OA.53}$$

where the *debt capacity factor* is

$$\Lambda(\beta,\rho,\xi,G) \;\equiv\; \mathbb{E}_0\left[\sum_{a=0}^{\hat{a}(\tilde{z}^\infty)-1} (\beta\rho)^a\, g\big(\Theta_a(\tilde{z}^a)\big)\, \tilde{z}_a^\theta + \sum_{a=\hat{a}(\tilde{z}^\infty)}^{T(\tilde{z}^\infty)-1} (\beta\rho)^a\, \tilde{z}_a^\theta + (\beta\rho)^{T(\tilde{z}^\infty)}\, \gamma(\tilde{z}^\infty)\, \tilde{z}_T^\theta\right]. \tag{OA.54}$$

The scalar $\Lambda(\beta,\rho,\xi,G)$ depends only on (β,ρ,ξ,G): it does not depend on χ, K, w, p_k, or r_f.

Deterministic special case. When $\tilde{z}_a = 1$ a.s., $\hat{a}$ and T are non-random and $g(\Theta_a)$ is evaluated at the deterministic multiplier (OA.47). Equation (OA.54) reduces to

$$\Lambda^{\text{det}} \;=\; \sum_{a=0}^{\hat{a}-1}(\beta\rho)^a\, g(\Theta_a^{\text{det}}) + \sum_{a=\hat{a}}^{T-1}(\beta\rho)^a + (\beta\rho)^T\gamma. \tag{OA.55}$$

OA.3.5.2 The participation constraint and optimal scale

At entry the lender finances blueprint capacity K at cost $p_k(1+r_f)K$. The participation constraint binds:

$$L_0 \;=\; p_k(1+r_f)\,K. \tag{OA.56}$$

Substituting (OA.53) into (OA.56) and solving, using $1-\eta\alpha\theta = (1-\eta)\theta$:

$$K^*(\xi,\chi) \;=\; \left[\frac{\Lambda(\beta,\rho,\xi,G)\cdot\pi_u(1,1,1)\cdot\chi^\theta}{p_k(1+r_f)}\right]^{1/(1-\eta)\theta}. \tag{OA.57}$$

The outer exponent $1/((1-\eta)\theta) > 1$ is the scale amplifier: because revenue is concave in K while the financing cost is linear, a higher debt capacity Λ raises optimal scale more than proportionally.

First-best benchmark. When $\xi = 1$ the enforcement constraint never binds: $\Theta_a = 1$ and $g(\Theta_a) = 1$ for all a, so the ELC is empty ($\hat{a} = 0$) and no efficiency loss arises. The entrepreneur directly maximizes the NPV of profits minus the cost of capital:

$$\max_{K} \quad \left(1 + \frac{\beta\rho\,\mu}{1-\beta\rho}\right) \cdot \pi_u(1,1,1) \cdot \chi^{\theta}\, K^{\eta\alpha\theta} \;-\; p_k(1+r_f)\, K. \tag{OA.58}$$

The first-order condition yields the first-best debt capacity:

$$\Lambda^{FB} \;=\; \eta\alpha\theta \left(1 + \frac{\beta\rho\,\mu}{1-\beta\rho}\right), \tag{OA.59}$$

and the unconstrained scale $K_u(\chi)$ has the same functional form as (OA.57) with $\Lambda(\xi)$ replaced by Λ^{FB}.

Scale decomposition.

$$K^*(\xi,\chi) \;=\; K_u(\chi) \cdot \left(\frac{\Lambda(\xi)}{\Lambda^{FB}}\right)^{1/(1-\eta)\theta}. \tag{OA.60}$$

All dependence on χ, w, p_k, and r_f is absorbed into $K_u(\chi)$. For cross-country comparisons:

$$\frac{K^*(\xi_c)}{K^*(\xi_{c'})} \;=\; \left(\frac{\Lambda(\xi_c)}{\Lambda(\xi_{c'})}\right)^{1/(1-\eta)\theta}. \tag{OA.61}$$

OA.3.5.3 Equity value and short-term debt at entry

The entrepreneur's equity value at entry is $V_0 \equiv V_0(\tilde{z}^0)$ evaluated at the optimal scale K^*. From Corollary 2:

$$V_0 \;=\; \Gamma(\beta,\rho,\xi,G) \cdot \pi_u(1,1,1) \cdot \chi^{\theta}\, K^{*\,\eta\alpha\theta}, \tag{OA.62}$$

where the *equity factor* is

$$\Gamma(\beta,\rho,\xi,G) \;\equiv\; \mathbb{E}_0\!\left[(\beta\rho)^{T(\tilde{z}^{\infty})}(1-\gamma(\tilde{z}^{\infty}))\,\tilde{z}_T^{\theta} + \frac{(\beta\rho)^{T(\tilde{z}^{\infty})+1}}{1-\beta\rho}\,\mu\right]. \tag{OA.63}$$

Like Λ, the equity factor Γ depends only on (β,ρ,ξ,G) and is free of χ, K, and factor prices.

Short-term debt at entry is $S_0 = \Theta_0\, S_u(K^*,\chi,1)$. Normalizing $\tilde{z}_0 = 1$ and substituting (OA.19):

$$S_0 \;=\; \Theta_0(\beta,\rho,\xi,G) \cdot S_u(1,1,1) \cdot \chi^{\theta}\, K^{*\,\eta\alpha\theta}, \tag{OA.64}$$

where $S_u(1,1,1) = [\eta(1-\alpha)/(1+r_f)]^{\eta(1-\alpha)\theta}\, w^{-\eta(1-\alpha)\theta}$ collects the working-capital price terms.

The total funding gap. At age a, using $S_a = \Theta_a \, S_u(K^*, \chi, \tilde{z}_a)$:

$$\frac{S_a}{S_u(K_u, \chi, \tilde{z}_a)} \;=\; \Theta_a(\tilde{z}^a) \cdot \left(\frac{K^*}{K_u}\right)^{\eta\alpha\theta}. \tag{OA.65}$$

All talent, productivity, and price terms cancel. Substituting the scale decomposition (OA.60):

$$\frac{S_a}{S_u(K_u, \chi, \tilde{z}_a)} \;=\; \underbrace{\Theta_a(\tilde{z}^a)}_{\text{within-firm constraint}} \cdot \underbrace{\left(\frac{\Lambda(\xi)}{\Lambda^{FB}}\right)^{\eta\alpha/(1-\eta)}}_{\text{scale distortion}}. \tag{OA.66}$$

The within-firm constraint Θ_a varies across firm ages as repayment history relaxes the enforcement condition; it converges to 1 as the firm approaches maturity. The scale distortion is common to all firms in a given enforcement regime, regardless of age, talent, or realized productivity. It is pinned down entirely by Λ, which does not depend on permanent talent χ, factor prices (w, p_k, r_f), or the firm's scale K. Because every firm in a given country faces the same ξ, the scale distortion is invisible in domestic data: it affects all firms equally and can only be detected by comparing $\Lambda(\xi_c)/\Lambda(\xi_{c'})$ across enforcement regimes.

OA.3.5.4 Optimal debt capacity

The entrepreneur chooses blueprint capacity K to maximize entry value V_0 subject to the binding PCF (OA.56). By the PCF, K and Λ are in one-to-one correspondence, so the problem can be restated with Λ as the choice variable. Substituting (OA.57) into (OA.62), all dependence on χ, w, p_k, and r_f factors out:

$$\max_{\Lambda} \quad \Gamma(\Lambda) \cdot \Lambda^{\eta\alpha/(1-\eta)}, \tag{OA.67}$$

where $\Gamma(\Lambda)$ is the equity factor delivered by the optimal front-loaded contract with initial normalized debt Λ.

The key trade-off is between *scale* and *contract cost*. Higher Λ relaxes the PCF and permits larger K^* (captured by $\Lambda^{\eta\alpha/(1-\eta)}$), but it also lengthens the expected contract maturity and tightens the ELC, reducing $\Gamma(\Lambda)$.

Total surplus decomposition. Let $\mathcal{V} \equiv 1 + \beta\rho\,\mu/(1-\beta\rho)$ denote gross surplus per unit of $\pi_u(1,1,1) \cdot \chi^\theta K^{\eta\alpha\theta}$. The lender's claim and the entrepreneur's equity exhaust surplus net of profit losses during the constrained phase:

$$\Lambda + \Gamma(\Lambda) \ \leq \ \mathcal{V}, \tag{OA.68}$$

with equality if and only if $\xi = 1$. The gap $\mathcal{V} - \Lambda - \Gamma(\Lambda) \geq 0$ reflects the cumulative efficiency loss from operating at $\Theta_a < 1$ during the ELC; it is strictly positive whenever $\xi < 1$ and increases in Λ because higher debt capacity lengthens the constrained phase. Differentiating (OA.68):

$$\Gamma'(\Lambda) \ \leq \ -1, \tag{OA.69}$$

with equality in the first best.

First-order condition. Differentiating (OA.67) with respect to Λ and dividing by $\Lambda^{\eta\alpha/(1-\eta)-1}$:

$$\frac{\eta\alpha}{1-\eta} \cdot \frac{\Gamma(\Lambda)}{\Lambda} \ = \ -\Gamma'(\Lambda) \ \geq \ 1. \tag{OA.70}$$

The left side is the marginal benefit of higher debt capacity: a larger Λ raises scale, and the resulting profit gain is proportional to $\eta\alpha/(1-\eta)$ times the equity-to-debt ratio Γ/Λ. The right side is the marginal cost: each unit of Λ transfers one unit to the lender and reduces the entrepreneur's equity by more than one unit because of the additional profit losses during the tighter ELC. Equation (OA.70) is an implicit equation for $\Lambda^*(\xi)$ as a function of $(\beta, \rho, \xi, \eta, \alpha, G)$; it does not involve χ, K, or factor prices. All entrepreneurs in a given enforcement regime choose the same $\Lambda^*(\xi)$, regardless of talent.

OA.3.5.5 Balance-sheet moments and identification

The scale distortion $K^*/K_u = (\Lambda^*/\Lambda^{FB})^{1/((1-\eta)\theta)}$ is governed by $\Lambda^*(\xi)$. This section identifies which observable moments recover Λ^*.

Invariance of Λ^*. The optimality condition (OA.70) determines Λ^* through $\Gamma(\Lambda;\xi)$, which depends only on $(\beta, \rho, \xi, \eta, \alpha, G)$. In particular,

$$\Lambda^*(\xi) \quad \text{does not depend on} \quad \chi,\ w,\ p_k,\ r_f,\ \text{or } K. \tag{OA.71}$$

Any data moment used to recover Λ^* must share this invariance.

Common-factor structure. Comparing (OA.19) and (OA.23):

$$S_u(1,1,1) \;=\; \frac{1}{1-\eta(1-\alpha)}\,\pi_u(1,1,1) \;=\; \theta\,\pi_u(1,1,1). \tag{OA.72}$$

Every entry-age balance-sheet object is a scalar multiple of $\mathcal{C} \equiv \pi_u(1,1,1)\cdot\chi^{\theta}\,K^{*\,\eta\alpha\theta}$:

$$L_0 \;=\; \Lambda^*\cdot\mathcal{C}, \qquad V_0 \;=\; \Gamma\cdot\mathcal{C}, \qquad S_0 \;=\; \theta\,\Theta_0\cdot\mathcal{C}. \tag{OA.73}$$

In any within-firm ratio, $\mathcal{C}$ cancels exactly.

Entry-age ratios: two degrees of freedom. The independent entry-age ratios are

$$\frac{V_0}{L_0} \;=\; \frac{\Gamma}{\Lambda^*}, \qquad \frac{S_0}{L_0} \;=\; \frac{\theta\,\Theta_0}{\Lambda^*}. \tag{OA.74}$$

All other entry-age leverage ratios are functions of these two. Entry-age data therefore provides exactly two independent equations in the three unknowns $(\Lambda^*,\Gamma,\Theta_0)$: the level of Λ^* is absorbed into $\mathcal{C}$ and cannot be recovered from ratios at a single age.

Why cross-age moments are necessary. At the ELC exit age $a=\hat{a}$, $\Theta_{\hat{a}}=1$, which eliminates one unknown and breaks the rank deficiency. Short-term debt at $\hat{a}$ is

$$S_{\hat{a}}(\tilde{z}^{\hat{a}}) \;=\; \theta\,\tilde{z}_{\hat{a}}^{\,\theta}\cdot\mathcal{C}. \tag{OA.75}$$

The cross-age ratio $S_{\hat{a}}/S_0$ isolates Θ_0:

$$\mathbb{E}\!\left[\frac{S_{\hat{a}}}{S_0}\right] \;=\; \frac{\mu_\theta}{\Theta_0}. \tag{OA.76}$$

Because $S_a = wn_a$, the balance-sheet version and the employment version coincide exactly, with the wage canceling.

Once Θ_0 is known, S_0/L_0 isolates Λ^*:

$$\Lambda^* \;=\; \frac{\theta\,\Theta_0}{S_0/L_0} \;=\; \frac{\theta\,\mu_\theta}{\mathbb{E}[S_{\hat{a}}/S_0]\cdot(S_0/L_0)}. \tag{OA.77}$$

The consolidated R moment. Define

$$R \;\equiv\; \frac{L_0}{S_{\hat a}} \;=\; \frac{\Lambda^*}{\theta\,\tilde z_{\hat a}^{\,\theta}}. \tag{OA.78}$$

In expectation:

$$\mathbb{E}[R] \;=\; \frac{\Lambda^*(\xi)}{\theta\,\mu_\theta}. \tag{OA.79}$$

Comparing two countries c and c':

$$\frac{\mathbb{E}[R_c]}{\mathbb{E}[R_{c'}]} \;=\; \frac{\Lambda^*(\xi_c)}{\Lambda^*(\xi_{c'})}, \tag{OA.80}$$

and the cross-country scale distortion is

$$\frac{K_c^*}{K_{c'}^*} \;=\; \left(\frac{\mathbb{E}[R_c]}{\mathbb{E}[R_{c'}]}\right)^{1/((1-\eta)\theta)}. \tag{OA.81}$$

Recovering the scale distortion from (OA.81) requires neither solving the dynamic program nor observing cross-country factor prices or the talent distribution.

Recovering the three contract scalars. Combining short-term debt growth, entry-age leverage, and the equity-to-debt ratio:

$$\Theta_0 = \frac{\mu_\theta}{\mathbb{E}[S_{\hat a}/S_0]}, \qquad \Lambda^* = \frac{\theta\,\Theta_0}{S_0/L_0}, \qquad \Gamma = \frac{V_0}{L_0}\cdot\Lambda^*. \tag{OA.82}$$

Age profiles as further restrictions. Short-term debt growth between consecutive ages during the ELC:

$$\frac{S_{a+1}}{S_a} \;=\; \frac{\Theta_{a+1}}{\Theta_a}\cdot\tilde z_{a+1}^{\,\theta}, \qquad \mathbb{E}\left[\frac{S_{a+1}}{S_a}\right] \;=\; \mu_\theta\,\frac{\Theta_{a+1}}{\Theta_a}. \tag{OA.83}$$

The entire path $\{\Theta_0,\Theta_1,\ldots,\Theta_{\hat a-1},1\}$ is identified from age profiles of short-term debt. Leverage profiles ℓ_a^S and ℓ_a^L by age provide additional overidentifying restrictions.

OA.3.6 Cross-sectional aggregation and measurement

Proposition OA.10 (Aggregation) *Let $S_a = \Theta_a\,S_u(K,\chi,\tilde z_a)$ with $S_u \propto \chi^\theta \tilde z_a^\theta K^{\eta\alpha\theta}$, and let $\hat a$ denote the reference age at which $\Theta_{\hat a} = 1$.*

(i) The ratio-of-means estimator $\Theta_a^{RM} \equiv \mathbb{E}[S_a]/\mathbb{E}[S_{\hat a}]$ satisfies $\Theta_a^{RM} = \mathbb{E}[\Theta_a\cdot\tilde z_a^\theta]/\mu_\theta$, where

$\mu_\theta \equiv \mathbb{E}[\tilde{z}^\theta]$. *Permanent talent* χ_i*, entry scale* K_i*, and factor prices cancel.*

(ii) The mean-of-ratios estimator $\Theta_a^{MR} \equiv \mathbb{E}[S_a/S_{\hat{a}}]$ *satisfies* $\Theta_a^{MR} = \Theta_a^{RM} \cdot \nu$*, where* $\nu \equiv \mu_\theta \, \mathbb{E}[\tilde{z}^{-\theta}] > 1$.

Short-term debt at age a is $S_a = \Theta_a \, S_u(K, \chi, \tilde{z}_a)$, where $S_u \propto \chi^\theta \tilde{z}_a^\theta K^{\eta\alpha\theta}$ by (OA.19). At the reference age $\hat{a}$ where $\Theta_{\hat{a}} = 1$, $S_{\hat{a}} = S_u$. Two cross-sectional estimators of the constraint multiplier follow.

Ratio-of-means estimator. Define

$$\Theta_a^{RM} \equiv \frac{\mathbb{E}[S_a]}{\mathbb{E}[S_{\hat{a}}]}. \tag{OA.84}$$

Since $S_a \propto \chi^\theta K^{\eta\alpha\theta}$ and both terms share the same factor, χ-invariance (Corollary 3) implies that all permanent firm characteristics and factor prices cancel. What remains is

$$\Theta_a^{RM} = \frac{\mathbb{E}[\Theta_a \cdot \tilde{z}_a^\theta]}{\mu_\theta}, \tag{OA.85}$$

where $\mu \equiv \mathbb{E}[\tilde{z}^\theta]$. Because $n_i = S_i/w$, the ratio $\mathbb{E}[S_a]/\mathbb{E}[S_{\hat{a}}]$ equals the ratio of aggregate employment at age a to aggregate employment at $\hat{a}$, so $1 - \Theta_a^{RM}$ is the fraction of aggregate young-firm employment missing at age a relative to the unconstrained benchmark.

Mean-of-ratios estimator. Define

$$\Theta_a^{MR} \equiv \mathbb{E}\left[\frac{S_a}{S_{\hat{a}}}\right]. \tag{OA.86}$$

Within a firm, talent cancels in the ratio $S_a/S_{\hat{a}}$, so the mean of ratios does not require a balanced cross section. The two estimators satisfy

$$\Theta_a^{MR} = \Theta_a^{RM} \cdot \nu, \qquad \nu \equiv \mu_\theta \cdot \mathbb{E}[\tilde{z}^{-\theta}] > 1. \tag{OA.87}$$

Under lognormality, $\nu = \exp(\theta^2\sigma^2)$. The inflation arises from Jensen's inequality applied to $\tilde{z}_{\hat{a}}^{-\theta}$. For cross-country comparison, the ratio of means is preferred: if σ differs across countries, ν varies and the mean of ratios conflates enforcement with dispersion effects. The observable diagnostic $\hat{\nu} = \Theta_a^{MR}/\Theta_a^{RM}$ tests whether dispersion heterogeneity is empirically relevant.

OA.3.7 Rebinding

Rebinding occurs when enforcement binds at some $a > \hat{a}$ for a firm that previously exited the ELC. From (OA.46), the firm is constrained when

$$\frac{(1-\eta(1-\alpha))\,\mu_\theta\,M_a}{(1-\xi)(1-\beta\rho)\,\tilde{z}_a^\theta} < 1. \tag{OA.88}$$

After ELC exit, M_a is high (debt nearly retired). Rebinding requires a sufficiently large $\tilde{z}_a$ to push the LHS below unity.

Lemma 5 (Sufficient condition for no rebinding) *Under bounded support $[\underline{z}, \bar{z}]$, there exists $z^* \in (\underline{z}, \bar{z})$ such that if $\tilde{z}_{\hat{a}} \leq z^*$, then $\Theta_a = 1$ for all $a > \hat{a}$.*

Proof. At ELC exit, $(1-\eta(1-\alpha))\,\mu\,M_{\hat{a}}/[(1-\xi)(1-\beta\rho)\,\tilde{z}_{\hat{a}}^\theta] = 1$. For $a > \hat{a}$, remaining debt declines and $M_a \geq M_{\hat{a}}$. Rebinding requires $\tilde{z}_a^\theta > \tilde{z}_{\hat{a}}^\theta$. Under bounded support with $\tilde{z}_a \leq \bar{z}$, firms exiting with $\tilde{z}_{\hat{a}}$ below a threshold z^* cannot rebind. ■

Rebinding requires a firm that exited the ELC with a moderate shock to subsequently draw $\tilde{z}_a > \tilde{z}_{\hat{a}}$. Under lognormal productivity with moderate dispersion, such sequences are rare.

Proposition OA.11 (Aggregation robustness) *If $|\ell_a^S| \leq \bar{\ell}$ a.s. and $\mathbb{P}(\mathcal{R}_a) \leq \varepsilon$, then*

$$\left|\mathbb{E}[\ell_a^S] - \mathbb{E}[\ell_a^S \mid \mathcal{R}_a^c]\right| \leq 2\bar{\ell}\,\varepsilon. \tag{OA.89}$$

Proof. Decompose $\mathbb{E}[\ell_a^S] = \mathbb{E}[\ell_a^S \mid \mathcal{R}_a^c]\mathbb{P}(\mathcal{R}_a^c) + \mathbb{E}[\ell_a^S \mid \mathcal{R}_a]\mathbb{P}(\mathcal{R}_a)$. Rearranging, $|\mathbb{E}[\ell_a^S] - \mathbb{E}[\ell_a^S \mid \mathcal{R}_a^c]| = |(\mathbb{E}[\ell_a^S \mid \mathcal{R}_a] - \mathbb{E}[\ell_a^S \mid \mathcal{R}_a^c])|\mathbb{P}(\mathcal{R}_a) \leq 2\bar{\ell}\varepsilon$. ■

Under lognormal productivity with moderate dispersion, the bias from ignoring rebinding is proportionally small.

OA.4 Steady-State Funding Gap Index

This appendix formalizes the funding gap index used in Section 6. It derives the stationary aggregation weights, states the index definition and decomposition cited in the main text, establishes identification, and extends the measure to employment units.

OA.4.1 From Funding Gaps to Employment Gaps

This section derives the employment decomposition stated in equation (17). The static first-order condition for labour, given Cobb-Douglas technology $y = (\chi_i \tilde{z}_a)^\eta (K^\alpha n^{1-\alpha})^\eta$, is

$$\eta(1-\alpha)\frac{y}{n} = (1+r_f)\, w. \tag{OA.90}$$

Solving for unconstrained employment yields

$$n_u(K, \chi_i, \tilde{z}_a) = \left[\frac{\eta(1-\alpha)}{(1+r_f)\, w}\right]^\theta (\chi_i\, \tilde{z}_a)^\theta\, K^{\eta\alpha\theta}, \tag{OA.91}$$

where $\theta = 1/[1-\eta(1-\alpha)]$. When the enforcement constraint binds, $n_a = \Theta_a \cdot n_u(K, \chi_i, \tilde{z}_a)$. For a firm of age a in economy c:

$$n_a^c = \Theta_a^c \cdot \left[\frac{\eta(1-\alpha)}{(1+r_f^c)\, w^c}\right]^\theta (\chi_i\, \tilde{z}_a)^\theta\, (K^c)^{\eta\alpha\theta}. \tag{OA.92}$$

Taking the ratio of (OA.92) across economies c and c' for the same $(\chi_i, \tilde{z}_a)$ gives

$$\frac{n_a^c}{n_a^{c'}} = \frac{\Theta_a^c}{\Theta_a^{c'}} \cdot \left(\frac{K^c}{K^{c'}}\right)^{\eta\alpha\theta} \cdot \left(\frac{w^{c'}}{w^c}\right)^\theta \cdot \left(\frac{1+r_f^{c'}}{1+r_f^c}\right)^\theta. \tag{OA.93}$$

The talent terms $(\chi_i\, \tilde{z}_a)^\theta$ cancel because both numerator and denominator are evaluated at the same $(\chi_i, \tilde{z}_a)$. This cancellation is an implication of degree-θ homogeneity (Proposition 7): all balance-sheet components and the labour demand function scale with χ_i^θ, so any ratio of the same object across enforcement regimes is talent-free.

Rewriting (OA.93) by grouping w and χ:

$$\frac{n_a^c}{n_a^{c'}} = \underbrace{\frac{\Theta_a^c}{\Theta_a^{c'}} \cdot \left(\frac{K^c}{K^{c'}}\right)^{\eta\alpha\theta}}_{\text{enforcement channel}} \times \underbrace{\left(\frac{w^{c'}/\chi^{c'}}{w^c/\chi^c}\right)^{\theta} \cdot \left(\frac{1+r_f^{c'}}{1+r_f^c}\right)^{\theta}}_{\text{effective unit labour cost}}. \tag{OA.94}$$

The object w^c/χ^c is the cost of labour per unit of entrepreneurial talent. The FOC (OA.90) implies that each firm's labour share equals $\eta(1-\alpha)/(1+r_f)$, independent of χ_i, K, and Θ. Because this property holds firm by firm, it aggregates: the economy-wide unit labour cost $w^c N^c/Y^c$ equals $\eta(1-\alpha)/(1+r_f^c)$ regardless of the talent distribution $F^c(\chi)$. In our data, unit labour costs are approximately 0.57 in both enforcement groups (Table 12), consistent with common technology parameters. The near-equalization (ratio ≈ 1.04) implies that the effective unit labour cost ratio in (OA.94) is close to unity.