

# Asset Liquidity and International Portfolio Choice

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## ABSTRACT

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We study optimal portfolio choice in a two-country model where assets represent claims on future consumption and facilitate trade in markets with imperfect credit. Assuming that foreign assets trade at a cost, agents hold relatively more domestic assets. Consequently, agents have larger claims to domestic over foreign consumption. Moreover, foreign assets turn over faster than domestic assets because the former have desirable liquidity properties, but represent inferior saving tools. Our mechanism offers an answer to a long-standing puzzle in international finance: a positive relationship between consumption and asset home bias coupled with higher turnover rates of foreign over domestic assets.

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# 1 Introduction

Agents' international portfolio choices have been a major topic of research in macroeconomics over the past two decades. The vast majority of papers emphasize the role that foreign assets play in helping agents diversify domestic income risk. While the role of assets in hedging risk is admittedly crucial, an equally important characteristic of assets, their liquidity, has been overlooked by the literature. In this paper, we develop a multi-country model that features a precise notion of asset liquidity, and we use it to reconcile a long-standing puzzle in international finance as described in Tesar and Werner (1995) and Lewis (1999): a positive relationship between consumption and asset home bias coupled with higher turnover rates of foreign over domestic assets. More precisely, developed countries obey three empirical regularities. First, in the average OECD country, agents' portfolios are heavily biased toward domestic assets. Second, economies that exhibit higher asset home bias also consume more domestic goods. Third, turnover rates of foreign assets are significantly higher than those of domestic assets.

To provide a concrete concept of asset liquidity, we employ a model in the tradition of monetary-search theory, extended to include real assets and trade between countries. Agents have access to alternating rounds of centralized, or Walrasian, markets and decentralized markets, where trade occurs in a bilateral fashion and credit is imperfect. In each country's Walrasian market, domestic and foreign agents can buy assets of that country at the ongoing market price. More interestingly, agents can bring a portfolio of assets to each country's decentralized market and trade it for locally-produced goods. Hence, assets serve a double function. First, they are claims to future consumption, as is standard in finance. Second, they serve as media of exchange, as is standard in monetary theory. It is precisely this second function that captures the notion of asset liquidity.

Within this framework, we characterize equilibria in which different assets arise as media of exchange in different types of bilateral meetings. Assuming that the net dividend from foreign assets is lower (e.g. due to a higher cross-border relative to domestic dividend income tax), domestic assets represent superior stores of value. Hence, in any exchange that involves only domestic agents, local assets serve as means of payment. An interesting tradeoff arises when agents choose which assets to bring to the foreign decentralized market to acquire consumption goods. Foreign assets have greater purchasing power in that market (because they are local to the producer), but market participation is not guaranteed. If agents' trading opportunities abroad are sufficiently frequent, then assets circulate as media of exchange locally; i.e. domestic assets facilitate trade at home and foreign assets facilitate trade abroad, thus endogenizing the common "currency area" assumption in international macroeconomics. At the other extreme, if trading opportunities abroad are scarce, agents use their domestic assets to acquire consumption goods both at home and abroad. Finally, in an intermediate case, either type of equilibrium described above can arise, depending on other parameters that govern asset returns.

To understand why our model can account for the three stylized facts, consider the empirically relevant “currency area” regime. Since domestic assets yield a higher dividend, as long as trading opportunities abroad are not much more frequent than at home, agents hold more domestic over foreign assets. Hence, portfolios exhibit home bias. In turn, agents have larger claims to domestic over foreign consumption goods. Hence, our suggested mechanism positively links asset and consumption home bias. Finally, agents hold foreign assets in order to acquire consumption abroad, but they offload them at the first given opportunity. Hence, foreign assets turn over faster than domestic assets. This result is very particular to our mechanism. In the absence of the liquidity channel, the dividend differential between domestic and foreign assets (or a similar per unit trading cost), which is necessary to generate consumption and asset home bias, would yield lower turnover of foreign relative to domestic assets. Therefore, the unique aspect of our model is the ability to capture the three stylized facts simultaneously.

We use the tight link between consumption and asset home bias to assess the model’s quantitative ability to explain equity holding and turnover rate patterns observed in the data. We calibrate the model’s key parameters to match the import-to-GDP ratio, domestic equity turnover rate, and difference in dividend income taxes between foreign (OECD) and home equities for the US over the past decade. We then let the model endogenously generate asset portfolios and turnover rates of foreign assets. The calibrated model predicts a home equity share of seventy-two percent and a turnover rate for foreign equities that is 1.44 times higher than the rate for domestic equities. These results favor well with the data. First, we document that US citizens have held eighty-five percent of their wealth in domestic equities during the 1997-2007 period. Second, Amadi and Bergin (2008) document that the turnover rate of foreign equities is 1.89 times as high as the domestic turnover rate for the US during the 1993-2001 period.<sup>1</sup>

This paper relates to a large literature that attempts to understand equity holdings and flows across borders. Heathcote and Perri (2007) use a standard international business cycle framework and show that, when preferences are biased toward domestic goods, asset home bias arises because endogenous international relative price fluctuations make domestic stocks a good hedge against non-diversifiable labor income risk. Their model generates a tight link between a country’s degree of openness to trade and level of asset diversification. This positive link is also explored by Collard, Dellas, Diba, and Stockman (2009) in an endowment economy with separable utility between traded and non-traded goods. In their model, an agent’s optimal portfolio includes the entire stock of home firms that produce domestic non-traded goods and a fully diversified portfolio of equities of firms that produce tradable goods. The authors show that, if the share of non-traded goods in consumption is large, the model can generate substantial portfolio home bias. Hnatkovska (2010) demonstrates that asset home bias and a

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<sup>1</sup> In Table 1, we document that equity home bias for the average OECD economy is seventy-four percent during the same decade. Amadi and Bergin (2008) document that the turnover rate of foreign equities is at least twice as high as the domestic turnover rate in Canada, Germany, and Japan during the same period.

high foreign asset turnover rate can arise in the presence of non-diversifiable non-traded consumption risk when each country specializes in production, preferences exhibit consumption home bias, and asset markets are incomplete. However, in this setting, domestic asset flows are also high, thus yielding equally high turnover rates of domestic assets.

Unlike the consumption-based models of home bias above, Amadi and Bergin (2008) offer an explanation for the coexistence of asset home bias and a higher turnover rate of foreign over domestic assets in a portfolio-choice model. Their mechanism relies on the existence of heterogeneous per-unit trading costs and homogenous fixed entry costs. While this mechanism may be in part responsible for the observations in the data, we argue that there is considerable room for an explanation that builds on a general-equilibrium consumption-based model like ours. First, unlike Amadi and Bergin's (2008), in addition to capturing the asset turnover rate differentials, our model links consumption and asset home bias, which is a regularity observed in the data. Second, in Section 7.2, we show evidence that more than half of cross-border portfolio investments are made by or on the behalf of individual investors in a typical developed economy such as the EU.

This paper is conceptually close to Lagos (2010). The author revisits the equity premium and risk-free rate puzzles through the lens of a model in which assets are valued as claims to streams of consumption and for their ability to facilitate trade. He shows that risk-averse agents are willing to hold the risk-free asset despite its lower yield, not only because this asset is safe, but also because it endogenously arises as a superior medium of exchange. Thus, Lagos's (2010) liquidity channel offers a new means to rationalize standard asset-pricing puzzles in the finance literature. In similar spirit, the present paper is the first to address a standard allocation puzzle in international finance through a liquidity channel.

Our paper falls into a growing literature that focuses on the liquidity properties of objects other than fiat money. Lagos and Rocheteau (2008) assume that part of the economy's physical capital can be used as a medium of exchange along with money. Their goal is to study the issue of over-investment and how it is affected by inflation. Geromichalos, Licari, and Suarez-Lledo (2007) study the coexistence of money and a real financial asset as media of exchange, with special focus on the relationship between asset prices and monetary policy. Lester, Postlewaite, and Wright (2012) take this framework a step further and endogenize the acceptability (of various media of exchange) decisions of agents.

Lastly, our paper relates to the literature of money-search models applied to international frameworks. In their pioneering work, Matsuyama, Kiyotaki, and Matsui (1993) employ a two-country, two-currency money-search model, with indivisible money and goods, and study conditions under which the two currencies serve as media of exchange in different countries. Wright and Trejos (2001) maintain the assumption of good indivisibility in Matsuyama *et al*, but they endogenize prices using bargaining theory. Head and Shi (2003) also consider a two-country, two-currency search model and show that the nominal exchange rate depends

on the stocks and growth rates of the two monies. Furthermore, Camera and Winkler (2003) use a search-theoretic model of monetary exchange in order to show that the absence of well-integrated international goods markets does not necessarily imply a violation of the law of one price. To the best of our knowledge, this is the first paper in which assets, other than fiat money, are endowed with liquidity properties, thus allowing us to bring the money-search literature closer to questions related to international portfolio diversification.

It should be pointed out that in Wright and Trejos (2001) and Head and Shi (2003) all valued currencies are “international” currencies, i.e. they serve as media of exchange in all types of decentralized meetings. In contrast, our paper delivers clear implications regarding which assets serve as means of payment in each market. For instance, in our “currency area” equilibrium, assets facilitate trade only in their country of origin. This is true not because (local) sellers do not accept foreign assets as means of payment, but rather because buyers rationally choose to not bring their home assets in order to trade in the decentralized market abroad. Hence, the paper’s ability to give rise to a “currency area” equilibrium is a contribution, not only to the international macroeconomics, but also to the monetary-search literature.

The remainder of the paper is organized as follows. Section 2 describes the modeling environment. Section 3 discusses the optimal behavior of agents in the economy. Section 4 characterizes the media of exchange that arise in a symmetric two-country model of international trade in goods and assets. Section 5 describes the model’s predictions regarding asset and consumption home bias as well as asset turnover rates, and it documents the three stylized facts. Section 6 evaluates the quantitative ability of the liquidity mechanism to account for the observations in the data. Section 7 discusses the robustness of the theoretical and quantitative results. Finally, Section 8 concludes.

## 2 Physical Environment

Time is discrete with an infinite horizon. Each period consists of two sub-periods. During the first sub-period, trade occurs in decentralized markets (*DM* henceforth), which we describe in detail below. In the second sub-period, economic activity takes place in traditional Walrasian or centralized markets (*CM* henceforth). There is no aggregate uncertainty. There are two countries, *A* and *B*. Each country has a unit measure of buyers and a measure  $\xi$  of sellers who live forever. The identity of agents (as sellers or buyers) is permanent. During the first sub-period, a distinct *DM* opens within each country and anonymous bilateral trade takes place. We refer to these markets as  $DM_i$ ,  $i = A, B$ . Without loss of generality, assume that  $DM_A$  opens first. Sellers from country *i* are immobile, but buyers are mobile. Therefore, in  $DM_i$ , sellers who are citizens of country *i* meet buyers who could be citizens of either country. During the second sub-period, all agents are located in their home country.

All agents discount the future between periods (but not sub-periods) at rate  $\beta \in (0, 1)$ . Buyers consume in both sub-periods and supply labor in the second sub-period. Their preferences, which are independent of their citizenship, are given by  $\mathcal{U}(q_A, q_B, X, H)$ , where  $q_i$  is consumption in  $DM_i$ ,  $i = A, B$ , and  $X, H$  are consumption and labor in the (domestic)  $CM$ . Sellers consume only in the  $CM$  and they produce in both the  $DM$  and the  $CM$ . Sellers' preferences are given by  $\mathcal{V}(h, X, H)$ , where the only new variable,  $h$ , stands for hours worked in the  $DM$ . In line with Lagos and Wright (2005), we adopt the functional forms

$$\begin{aligned}\mathcal{U}(q_A, q_B, X, H) &= u(q_A) + u(q_B) + U(X) - H, \\ \mathcal{V}(h, X, H) &= -c(h) + U(X) - H.\end{aligned}$$

We assume that  $u$  and  $U$  are twice continuously differentiable with  $u(0) = 0$ ,  $u' > 0$ ,  $u'(0) = \infty$ ,  $U' > 0$ ,  $u'' < 0$ , and  $U'' \leq 0$ . For simplicity, we set  $c(h) = h$ , but this is not crucial for any of our results. Let  $q^* \equiv \{q : u'(q) = 1\}$ , i.e.  $q^*$  denotes the optimal level of production in any bilateral meeting. Also, suppose that there exists  $X^* \in (0, \infty)$  such that  $U'(X^*) = 1$ , with  $U(X^*) > X^*$ .

During the round of decentralized trade, sellers and buyers are matched randomly according to a matching technology that is identical in both  $DM$ 's. Let  $\mathcal{B}, \mathcal{S}$  denote the total number of buyers and sellers in a certain  $DM$ .<sup>2</sup> The total number of matches in this market is given by  $M(\mathcal{B}, \mathcal{S}) \leq \min\{\mathcal{B}, \mathcal{S}\}$ , where  $M$  is increasing in both arguments. Since only the aggregate measure of buyers (as opposed to the individual measures of local and foreign buyers) appears in  $M$ , the matching technology is unbiased with respect to the citizenship of the buyer. The arrival rate of buyers to an arbitrary seller is  $a_s = M(\mathcal{B}, \mathcal{S})/\mathcal{S}$ , and the arrival rate of sellers to an arbitrary buyer is  $a_b = M(\mathcal{B}, \mathcal{S})/\mathcal{B}$ . In both  $DM$ 's,  $\mathcal{S} = \xi$ . Buyers get to visit the domestic  $DM$  with probability  $\sigma_H \in (0, 1)$  and the foreign  $DM$  with probability  $\sigma_F \in (0, 1)$ , so that in both  $DM$ 's  $\mathcal{B} = \sigma_H + \sigma_F$ . The relative magnitude of  $\sigma_F$  to  $\sigma_H$  captures the degree of economic integration. In any bilateral meeting, the buyer makes a take-it-or-leave-it offer to the seller. Any sale to a citizen of country  $j$  will count as imports of country  $j$  from country  $i$ .

As mentioned earlier, during the second sub-period agents trade in centralized markets,  $CM_i$ ,  $i = A, B$ . All agents consume and produce a general good or fruit which is identical in both countries. Thus, the domestic and the foreign general goods enter as perfect substitutes in the utility function. Agents are located in the home country and have access to a technology that can transform one unit of labor into one unit of the fruit. Furthermore, we assume that there are two trees, one in each country, that produce fruit, as in Lucas (1978). Shares of the tree in country  $i$  are traded in  $CM_i$ , but due to perfect financial integration, agents from country  $j$  can place any order and buy shares of this tree at the ongoing price  $\psi_i$ . Let  $T_i > 0$  denote the total supply of the tree in country  $i$  and  $d_i$  the per-period dividend of tree  $i$ . Since in this paper

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<sup>2</sup>To avoid the possibility of confusion, variables indexed by the term  $B$  are related to "country  $B$ ". On the other hand, variables indexed by the term  $\mathcal{B}$  are related to "Buyers".

we focus on symmetric equilibria, we assume that  $T_A = T_B = T > 0$  and  $d_A = d_B = d > 0$ .  $T$  and  $d$  are exogenously given and constant.

Except from consuming the general good and trading shares of the trees in the  $CM$ 's, buyers can also carry some claims into the decentralized round of trade and trade them for the good produced by local sellers. Hence, assets serve not only as stores of value, but also as media of exchange. The necessity for a medium of exchange arises due to anonymity and a double coincidence of wants problem that characterizes trade in the  $DM$ 's (see Kocherlakota (1998) for an extensive discussion). Assets can serve as media of exchange as long as they are portable, storable, divisible, and recognizable by all agents. We assume that all these properties are satisfied. As we explain in more detail below, we do not place any *ad hoc* restrictions on which assets can serve where as means of payments.

In the absence of any frictions in the physical environment, due to the symmetry assumption made earlier, the model would predict that agents' share of foreign assets in their portfolios is anywhere between 0 and 100%. In order to derive sharper predictions, additional assumptions are necessary. One friction that is very common in the international macroeconomics literature is the so-called *currency area* assumption, which dictates that trading in country  $i$  requires the use of that country's currency (in our case the asset) as a medium of exchange. This assumption is extremely appealing and empirically relevant. However, since our paper is in the spirit of modern monetary theory, we consider such a restriction undesirable, and we insist that agents should choose which assets to use as media of exchange.

We now introduce the main friction of our model. We assume that whenever an agent from country  $i$  holds one share of country  $j$ 's tree,  $j \neq i$ , she has a claim to  $d - \kappa$  units of fruit, with  $\kappa \in (0, d)$ . One can think of  $\kappa$  as a policy friction, such as a tax on foreign dividend income, which is commonly observed across many countries (this is precisely how we interpret  $\kappa$  in the quantitative exercise of Section 6). Alternatively,  $\kappa$  may capture an information friction or a cost that agents have to pay in order to participate in the foreign asset market.<sup>3</sup> Another way to think about  $\kappa$  follows from a more literal interpretation of the Lucas tree model: when an agent from country  $i$  holds one share of tree  $j$ ,  $d$  units of fruit (general good) have to be physically delivered from country  $j$  to its claimant in country  $i$ . Therefore,  $\kappa$  could also represent a transportation cost. It is important to highlight that all the theoretical results presented in this paper hold for arbitrarily small values of  $\kappa$ .

Finally, the notion of liquidity employed in this paper draws upon the recent monetary-search literature. Within that spirit, we assume that assets serve as means of payments in markets with imperfect credit.<sup>4</sup> However, our mechanism intends to capture the broader notion of assets as facilitators of trade. For example, sellers may require assets as collateral in order to

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<sup>3</sup> Similarly, Michaelides (2003) assumes that agents face a higher transaction cost when investing abroad.

<sup>4</sup> This is in contrast with the finance literature, which typically relates the liquidity of an asset to the speed with which it can be converted into consumption. See for example Duffie, Garleanu, and Pedersen (2005).

deliver goods to buyers.<sup>5</sup> Alternatively, agents may rely on repurchase agreements to acquire goods and services. As Lagos (2011) points out, contractual details aside, in all of these environments assets effectively act as media of exchange in that they facilitate trade between sellers and untrustworthy buyers.

### 3 Value Functions and Optimal Behavior

We begin with the description of the value functions in the *CM*. For a buyer from country  $i = A, B$ , the Bellman's equation is given by

$$W_i^B(t) = \max_{X, H, \hat{t}} \{U(X) - H + \beta V_i^B(\hat{t})\}$$

$$\text{s.t. } X + \psi_i (\hat{t}_{ii} + \hat{t}_{ij}) + \psi_j (\hat{t}_{ji} + \hat{t}_{jj}) = H + (\psi_i + d)(t_{ii} + t_{ij}) + (\psi_j + d_\kappa)(t_{ji} + t_{jj}).$$

The state variables are summarized by  $t \equiv (t_{ii}, t_{ij}, t_{ji}, t_{jj})$ , where  $t_{ij}$  is the amount of asset  $i$  that is used for trade in  $DM_j$ , and variables with hats denote next period's choices.<sup>6</sup> We have also defined  $d_\kappa = d - \kappa$ . It can be easily verified that, at the optimum,  $X = X^*$ . Using this fact and replacing  $H$  from the budget constraint into  $W_i^B$  yields

$$W_i^B(t) = U(X^*) - X^* + (\psi_i + d)(t_{ii} + t_{ij}) + (\psi_j + d_\kappa)(t_{ji} + t_{jj})$$

$$+ \max_{\hat{t}} \{-\psi_i (\hat{t}_{ii} + \hat{t}_{ij}) - \psi_j (\hat{t}_{ji} + \hat{t}_{jj}) + \beta V_i^B(\hat{t})\}. \quad (1)$$

Two observations are in order. First, since  $\mathcal{U}$  is quasi-linear, the optimal choice of  $\hat{t}$  does not depend on  $t$ , i.e. there are no wealth effects. Second,  $W_i^B$  is linear, and we can write

$$W_i^B(t) = \Lambda_i^B + (\psi_i + d)(t_{ii} + t_{ij}) + (\psi_j + d_\kappa)(t_{ji} + t_{jj}),$$

where the definition of  $\Lambda_i^B$  is obvious.

We now focus on sellers. As it is standard in monetary models, carrying assets across periods of time comes at a cost (in the case of fiat money, the cost is just the nominal interest rate). The only agents who are willing to incur this cost are the agents who benefit from the asset's liquidity, i.e. the buyers. Since in our model the identity of agents, as buyers or sellers, is fixed, sellers will always choose to leave the *CM* with zero asset holdings. For a more detailed proof

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<sup>5</sup> This notion of liquidity is closely related to the concept of pledgeability, which was introduced to the literature by Kiyotaki and Moore (1997) and Holmström and Tirole (1998).

<sup>6</sup> Hence, the buyer can bring any amount of local or foreign asset to the local  $DM$ , and the same is true for the foreign  $DM$ . This modeling choice guarantees that the order in which the  $DM$ 's open is inessential. If the buyer was choosing some  $(\hat{t}_i, \hat{t}_j)$ , which she could bring to either  $DM$ , the amount of assets that she would choose to trade in, say,  $DM_i$  would crucially depend on whether she got/will get matched in  $DM_j$ . That is, the buyer's behavior would depend on the order in which the  $DM$ 's open. We consider this an undesirable feature.

of this result, see Rocheteau and Wright (2005).<sup>7</sup> Noting that sellers also choose  $X = X^*$  in every period, we can write the value function of a seller from country  $i = A, B$  as

$$\begin{aligned} W_i^S(t_i, t_j) &= U(X^*) - X^* + (\psi_i + d) t_i + (\psi_j + d_\kappa) t_j + \beta V_i^S(0, 0) \\ &\equiv \Lambda_i^S + (\psi_i + d) t_i + (\psi_j + d_\kappa) t_j. \end{aligned}$$

Like in the buyers' case,  $W_i^S$  is linear. Sellers leave the  $CM$  with zero asset holdings, but they might enter this market with positive amounts of both assets, which they received as means of payments in the preceding  $DM$ .

We now turn to the terms of trade in the  $DM$ 's. As explained earlier, we place no restrictions on which assets can be used as media of exchange. Consider meetings in  $DM_i$ , and as a first case let the buyer be a citizen of country  $i$  (a local) with asset holdings denoted by  $t$ . The solution to the bargaining problem is a list  $(q_i, x_{ii}, x_{ji})$ , where  $q_i$  is the amount of special good,  $x_{ii}$  is the amount of asset  $i$ , and  $x_{ji}$  is the amount of asset  $j$  that changes hands. With take-it-or-leave-it offers by the buyer, the bargaining problem is

$$\begin{aligned} \max_{q_i, x_{ii}, x_{ji}} & [u(q_i) + W_i^B(t_{ii} - x_{ii}, t_{ij}, t_{ji} - x_{ji}, t_{jj}) - W_i^B(t)] \\ \text{s.t.} & -q_i + W_i^S(x_{ii}, x_{ji}) - W_i^S(0, 0) = 0 \\ & \text{and } x_{ii} \leq t_{ii}, x_{ji} \leq t_{ji}. \end{aligned} \tag{2}$$

Exploiting the linearity of the  $W$ 's, we can re-write this problem as

$$\begin{aligned} \max_{q_i, x_{ii}, x_{ji}} & [u(q_i) - (\psi_i + d)x_{ii} - (\psi_j + d_\kappa)x_{ji}] \\ \text{s.t.} & q_i = (\psi_i + d)x_{ii} + (\psi_j + d_\kappa)x_{ji} \\ & \text{and } x_{ii} \leq t_{ii}, x_{ji} \leq t_{ji}. \end{aligned}$$

The following lemma describes the bargaining solution in detail.

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<sup>7</sup>In Rocheteau and Wright (2005), the nominal interest rate is assumed to be strictly positive. Hence, holding no money is the unique optimal choice for sellers. In our model, there will be a case in which the cost of carrying the home asset is zero (this will never be the case for the foreign asset because of the friction  $\kappa$ ). In this case, sellers are indifferent between holding zero or some positive amount of the home asset. Nevertheless, holding zero asset is *always* optimal. Moreover, if one assumes that there is a cost,  $c > 0$ , of participating in the asset markets, then holding zero assets is the unique optimal choice for the seller, even for a tiny  $c$ .

**Lemma 1.** Define  $\pi_i \equiv (\psi_i + d)t_{ii} + (\psi_j + d_\kappa)t_{ji}$ .

$$\begin{aligned} \text{If } \pi_i \geq q^*, \quad \text{then} \quad & \begin{cases} q_i = q^*, \\ (\psi_i + d)x_{ii} + (\psi_j + d_\kappa)x_{ji} = q^*. \end{cases} \\ \text{If } \pi_i < q^*, \quad \text{then} \quad & \begin{cases} q_i = \pi_i, \\ x_{ii} = t_{ii}, x_{ji} = t_{ji}. \end{cases} \end{aligned}$$

*Proof.* It can be easily verified that the suggested solution satisfies the necessary and sufficient conditions for maximization.  $\square$

All that matters for the bargaining solution is whether the buyer's asset balances, i.e.  $\pi_i$ , are sufficient to buy the optimal quantity  $q^*$ . If the answer to that question is yes, then  $q_i = q^*$ , and the buyer spends amounts of assets  $t_{ii}, t_{ji}$  such that  $\pi_i = q^*$ . Notice that in this case  $t_{ii}, t_{ji}$  cannot be pinned down separately.<sup>8</sup> Conversely, if  $\pi_i < q^*$ , the buyer gives up all her asset holdings and purchases as much  $q$  as her balances allow.

Now consider a meeting in  $DM_i$  when the buyer is from  $j \neq i$  (a foreigner) with asset holdings  $t$ . Again, denote the solution by  $(q_i, x_{ii}, x_{ji})$ . The bargaining problem to be solved is the same as in (2), after replacing  $W_i^B$  with  $W_j^B$ . Using the linearity of the value functions, one can write the bargaining problem as

$$\begin{aligned} & \max_{q_i, x_{ii}, x_{ji}} [u(q_i) - (\psi_i + d_\kappa)x_{ii} - (\psi_j + d)x_{ji}] \\ \text{s.t.} \quad & q_i = (\psi_i + d)x_{ii} + (\psi_j + d_\kappa)x_{ji} \\ \text{and} \quad & x_{ii} \leq t_{ii}, x_{ji} \leq t_{ji}. \end{aligned}$$

Substituting for  $(\psi_i + d)x_{ii} + (\psi_j + d)x_{ji}$  from the constraint, we can re-write the objective as  $u(q_i) - q_i - \kappa x_{ji} + \kappa x_{ii}$ . Notice that for every unit of asset  $i$  that goes from the buyer to the seller, two positive effects are generated. First, trade is facilitated, i.e. the seller produces  $q$  for the buyer in exchange for the asset. Second, the social surplus increases because the asset goes to the hands of the agent who has a higher valuation for it. The next Lemma describes the solution to the bargaining problem in detail.

**Lemma 2.** Define  $\bar{q}(\psi) \equiv \left\{ q : u'(q) = \frac{\psi + d_\kappa}{\psi + d} \right\}$  and  $\underline{q}(\psi) \equiv \left\{ q : u'(q) = \frac{\psi + d}{\psi + d_\kappa} \right\}$ , with  $\bar{q}(\psi) > q^* > \underline{q}(\psi)$ , for all  $\psi < \infty$ . The bargaining solution is the following

$$a) \text{ If } t_{ii} \geq \frac{\bar{q}(\psi_i)}{\psi_i + d}, \text{ then } \begin{cases} q_i = \bar{q}(\psi_i), \\ x_{ii} = \frac{\bar{q}(\psi_i)}{\psi_i + d}, \\ x_{ji} = 0. \end{cases}$$

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<sup>8</sup> This does not cause any indeterminacy issues because, as we show later, buyers set  $t_{ji} = 0$ .

$$\begin{aligned}
b) \text{ If } t_{ii} \in \left[ \frac{q(\psi_j)}{\psi_i+d}, \frac{\bar{q}(\psi_i)}{\psi_i+d} \right), \text{ then } & \begin{cases} q_i = t_{ii}(\psi_i + d), \\ x_{ii} = t_{ii}, \\ x_j = 0. \end{cases} \\
c1) \text{ If } t_{ii} < \frac{q(\psi_j)}{\psi_i+d} \text{ and } \pi_i \geq \underline{q}(\psi_j), \text{ then } & \begin{cases} q_i = \underline{q}(\psi_j), \\ x_{ii} = t_{ii}, \\ x_{ji} = \frac{q(\psi_j) - (\psi_i+d)t_{ii}}{\psi_j+d\kappa}. \end{cases} \\
c2) \text{ If } t_{ii} < \frac{q(\psi_j)}{\psi_i+d} \text{ and } \pi_i < \underline{q}(\psi_j), \text{ then } & \begin{cases} q_i = \pi_i, \\ x_{ii} = t_{ii}, \\ x_{ji} = t_{ji}. \end{cases}
\end{aligned}$$

*Proof.* See Appendix A. □

The solution is very intuitive. The buyer, citizen of  $j$ , should use only asset  $i$  whenever possible. If her asset- $i$  holdings are unlimited, she should buy the quantity defined as  $\bar{q}(\psi)$ . This quantity is larger than  $q^*$ , the maximizer of  $u(q) - q$ , because of the second positive effect of using asset  $i$  described above (the wedge between the buyer's and the seller's valuation). In similar spirit, if the buyer's balances allow her to buy  $\underline{q}(\psi)$  or more, she should not use any amount of asset  $j$  for trade. Positive amounts of asset  $j$  will change hands, only if the asset- $i$  holdings are such that the buyer cannot purchase  $\underline{q}(\psi)$ . In that case, the buyer will use the amount of asset  $j$  that, together with all of her asset- $i$  holdings, buys the quantity  $\underline{q}(\psi)$ .

Our last task in this section is to discuss the optimal portfolio choice of the buyer. To that end, we first describe the buyer's value function in the  $DM$ . Once this function has been established, we can plug it into equation (1) and characterize this agent's objective function. Define  $p_H \equiv a_B \sigma_H$  and  $p_F \equiv a_B \sigma_F$ , i.e.  $p_H$  is the probability of matching in the local  $DM$ , and  $p_F$  is the analogous expression for the foreign  $DM$ . Then, the value function for a buyer from country  $i$  who enters the round of decentralized trade with asset holdings  $t = (t_{ii}, t_{ij}, t_{ji}, t_{jj})$ , is given by

$$\begin{aligned}
V_i^B(t) &= p_H p_F \left\{ u(q_i) + u(q_j) + W_i^B(t_{ii} - x_{ii}, t_{ij} - x_{ij}, t_{ji} - x_{ji}, t_{jj} - x_{jj}) \right\} \\
&+ p_H (1 - p_F) \left\{ u(q_i) + W_i^B(t_{ii} - x_{ii}, t_{ij}, t_{ji} - x_{ji}, t_{jj}) \right\} \\
&+ p_F (1 - p_H) \left\{ u(q_j) + W_i^B(t_{ii}, t_{ij} - x_{ij}, t_{ji}, t_{jj} - x_{jj}) \right\} \\
&+ (1 - p_H)(1 - p_F) W_i^B(t), \tag{3}
\end{aligned}$$

where  $q, x$  are determined by the bargaining protocols described in Lemmata 1 and 2. It is understood that  $(q_i, x_{ii}, x_{ji})$  are functions of  $(t_{ii}, t_{ji})$  and  $(q_j, x_{ij}, x_{jj})$  are functions of  $(t_{ij}, t_{jj})$ .

The next step is to lead equation (3) by one period in order to obtain  $V_i^B(\hat{t})$ . Then, plug the latter into (1) and focus only on the terms that contain the control variables  $\hat{t} = (\hat{t}_{ii}, \hat{t}_{ij}, \hat{t}_{ji}, \hat{t}_{jj})$ .

This is the objective function of the buyer from country  $i$ , and it can be written as

$$J_i^B(\hat{t}) = J_{i,H}^B(\hat{t}_{ii}, \hat{t}_{ji}) + J_{i,F}^B(\hat{t}_{ij}, \hat{t}_{jj}), \quad (4)$$

where we have defined

$$\begin{aligned} J_{i,H}^B(\hat{t}_{ii}, \hat{t}_{ji}) &= \left[ -\psi_i + \beta(\hat{\psi}_i + d) \right] \hat{t}_{ii} + \left[ -\psi_j + \beta(\hat{\psi}_j + d_\kappa) \right] \hat{t}_{ji} \\ &+ \beta p_H \left\{ u(q_i(\hat{t}_{ii}, \hat{t}_{ji})) - (\hat{\psi}_i + d) x_{ii}(\hat{t}_{ii}, \hat{t}_{ji}) - (\hat{\psi}_j + d_\kappa) x_{ji}(\hat{t}_{ii}, \hat{t}_{ji}) \right\}, \end{aligned} \quad (5)$$

$$\begin{aligned} J_{i,F}^B(\hat{t}_{ij}, \hat{t}_{jj}) &= \left[ -\psi_i + \beta(\hat{\psi}_i + d) \right] \hat{t}_{ij} + \left[ -\psi_j + \beta(\hat{\psi}_j + d_\kappa) \right] \hat{t}_{jj} \\ &+ \beta p_F \left\{ u(q_j(\hat{t}_{ij}, \hat{t}_{jj})) - (\hat{\psi}_i + d) x_{ij}(\hat{t}_{ij}, \hat{t}_{jj}) - (\hat{\psi}_j + d_\kappa) x_{jj}(\hat{t}_{ij}, \hat{t}_{jj}) \right\}. \end{aligned} \quad (6)$$

The term  $J_{i,n}^B$  is the part of the objective that reflects the choice of assets to be traded in  $DM_n$ , with  $n = H$  for home or  $n = F$  for foreign.

Equation (4) highlights that the optimal choice of asset holdings to be traded in the home and foreign  $DM$  ( $(\hat{t}_{ii}, \hat{t}_{ji})$  and  $(\hat{t}_{ij}, \hat{t}_{jj})$ , respectively) can be studied in isolation. The term  $-\psi_i + \beta(\hat{\psi}_i + d)$  represents the net gain from carrying one unit of the domestic asset from today's  $CM$  into tomorrow's  $CM$ . Sometimes we refer to the negative of this term as the cost of carrying the asset across periods. Similarly,  $-\psi_j + \beta(\hat{\psi}_j + d_\kappa)$  is the net gain from carrying one unit of the foreign asset across consecutive  $CM$ 's. In both (5) and (6), the second line represents the expected (discounted) surplus of the buyer in each  $DM$ . The following Lemma states an important result regarding the sign of the cost terms that appear in the agents' objective functions.

**Lemma 3.** *In any equilibrium,  $\psi_l \geq \beta(\hat{\psi}_l + d)$ ,  $l = A, B$ .*

*Proof.* This is a standard result in monetary theory. If  $\psi_l < \beta(\hat{\psi}_l + d)$  for some  $l$ , agents of country  $l$  have an infinite demand for this asset, and so equilibrium is not well defined.  $\square$

According to the Lemma, the net gain from carrying home assets across periods is non-positive and the net gain from carrying foreign assets across periods is strictly negative due to the term  $\kappa$ . This result assigns an intuitive interpretation to the objective function established above: a buyer wishes to bring assets with her in the  $DM$  in order to facilitate trade. However, she faces a trade-off because carrying these assets is not free (equations (5) and (6)).<sup>9</sup> We are now ready to discuss the optimal portfolio choice of the buyer and, consequently, equilibrium.

<sup>9</sup> Lemma 3 also clarifies our claim that it is always optimal for sellers to hold zero assets. In the case of foreign assets, this is the unique optimal choice.

## 4 Equilibrium in the Two-Country Model

We begin this section with a general definition of equilibrium and we explain why focusing on symmetric, steady-state equilibria can make the analysis more tractable.

**Definition 1.** An equilibrium for the two-country economy is a list of solutions to the bargaining problems in  $DM_i$ ,  $i = A, B$  described by Lemmata 1 and 2 and bounded paths of  $\psi_A, \psi_B$ , such that buyers maximize their objective function (described by (5) and (6)) under the market clearing conditions  $\sum_{l=A,B} \sum_{i=A,B} t_{Al}^i = T = \sum_{l=A,B} \sum_{i=A,B} t_{Bl}^i$ . The term  $t_{jl}^i$  denotes demand of a buyer from country  $i$  for asset  $j$  to trade in  $DM_l$ .

In the remainder of the paper we focus on symmetric, steady-state equilibria. In our model,  $p_H, p_F$  are the same in both countries. This fact, in combination with the strict concavity of  $J_{i,H}^B$  and  $J_{i,F}^B$  (which is standard in the Lagos-Wright model), implies the following: each buyer has a certain (degenerate) demand for the home asset and a certain (degenerate) demand for the foreign asset, but these demand functions do not depend on the agent's citizenship. In other words,  $t_{AA}^A = t_{BB}^B$ ,  $t_{AB}^A = t_{BA}^B$ ,  $t_{BA}^A = t_{AB}^B$ , and  $t_{BB}^A = t_{AA}^B$ .<sup>10</sup> Two important implications follow. First, both assets have equal aggregate supply ( $T$ ) and aggregate demand. Therefore, their equilibrium price has to be equal,  $\psi_A = \psi_B = \psi$ . Second, by the bargaining protocols, the amount of special good that changes hands in any  $DM$  depends only on whether the buyer is a local or a foreigner, but not on the label of the  $DM$ . These facts lead to the following definition.

**Definition 2.** A symmetric, steady-state equilibrium for the two-country economy can be summarized by the objects  $\{t_{HH}, t_{HF}, t_{FH}, t_{FF}, q_H, q_F, \psi\}$ . The term  $t_{ij}$  is the equilibrium asset holdings of the representative buyer (of any country) for asset  $i$  to be used for trade in  $DM_j$ , with  $i, j = H$  for home or  $i, j = F$  for foreign. For future reference also define the total home and foreign asset holdings of buyers,  $t_H = t_{HH} + t_{HF}$  and  $t_F = t_{FH} + t_{FF}$ . The term  $q_i$  stands for the amount of special good that changes hands in any  $DM$  when the buyer is local ( $i = H$ ) or foreign ( $i = F$ ). Finally,  $\psi$  is the symmetric, steady-state equilibrium asset price. Equilibrium objects are such that agents maximize their respective objective functions and markets clear.

We now proceed to a more careful discussion of the optimal portfolio choice of buyers and, consequently, equilibrium. Notice that the symmetric, steady-state version of Lemma 3 dictates that  $\psi \geq \beta d / (1 - \beta) \equiv \psi^*$ . The term  $\psi^*$  is the so-called *fundamental* value of the asset, i.e. the unique price that agents would be willing to pay for one unit of this asset if we were to shut down the  $DM$ 's (in which case the model would coincide with a two-country Lucas-tree

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<sup>10</sup> For example,  $t_{AA}^A = t_{BB}^B$  means that the demand for asset  $A$  of a buyer from  $A$  in order to trade in her local  $DM$  is equal to the demand of a buyer from  $B$  for asset  $B$  used for trade in her local  $DM$ . The remaining equations admit similar interpretations.

model). We will examine the choice of  $(t_{HH}, t_{FH})$  and  $(t_{HF}, t_{FF})$  separately, by looking at the symmetric, steady-state versions of (5) and (6).

**Lemma 4.** *A buyer's optimal choice of asset holdings for trade in the local DM satisfies  $t_{FH} = 0$ . Moreover, if  $\psi > \psi^*$ ,  $t_{HH}$  solves*

$$\psi = \beta(\psi + d) \{1 + p_H[u'((\psi + d)t_{HH}) - 1]\}, \quad (7)$$

and if  $\psi = \psi^*$ ,  $t_{HH} \geq q^*/(\psi + d)$ .

*Proof.* See Appendix A. □

Lemma 4 reveals that buyers never carry foreign assets to trade in the local DM. The intuition behind this result is straightforward. Recall from Lemma 1 that any combination of assets for which  $(\psi + d)t_{HH} + (\psi + d_\kappa)t_{FH} = \pi$ , for some given  $\pi$ , buys the same amount of special good. Since the foreign asset has a higher holding cost, it is always optimal for the buyer to purchase any desired quantity using  $t_{HH}$  only. When the cost to carry assets falls to zero, optimality requires that the buyer bring any amount of assets that buy her  $q^*$  in the local DM (this amount maximizes the buyer's surplus). By Lemma 1, any  $t_{HH} \geq (1 - \beta)q^*/d$  does that job.

Next, we consider the optimal choice of assets used for trade in the foreign DM. There are three scenarios (or regimes), that depend on the values of the following parameters:  $\beta$ ,  $p_F$ , and  $\kappa/d$ . In the first scenario, regardless of asset prices, the agent uses only foreign assets to trade in the foreign DM. In the second scenario, again regardless of asset prices, the agent chooses  $t_{FF} = 0$  and trades in the foreign DM with her home asset. Finally, there is a third regime, in which the agent uses either  $t_{HF}$  or  $t_{FF}$  as means of payment (except from a knife-edge case), depending on asset prices. The following Lemma describes the details. Figure 1 summarizes the parameter values that constitute the various regions described in Lemma 5.

**Lemma 5. Case 1:** *Assume  $(p_F, \beta) \in R_1$ , or  $(p_F, \beta) \in R_2$  and  $\frac{\kappa}{d} > \frac{(1-2p_F)}{(1-p_F)(1-\beta)} \equiv \tilde{\kappa}$ . Then the buyer uses only the foreign asset as a medium of exchange in the foreign DM. The optimal  $t_{FF}$  satisfies*

$$\psi = \beta [(1 - p_F)(\psi + d_\kappa) + p_F(\psi + d) u'((\psi + d)t_{FF})]. \quad (8)$$

If  $\psi > \psi^*$ , then  $t_{HF} = 0$ , and if  $\psi = \psi^*$ , then  $t_{HF} \in \mathbb{R}_+$ . Also,  $q_F = (\psi + d)t_{FF}$ .

**Case 2:** *Assume  $(p_F, \beta) \in R_4$ . The buyer sets  $t_{FF} = 0$  and uses only the domestic asset as a medium of exchange in the foreign DM. If  $\psi > \psi^*$ , then  $t_{HF}$  satisfies*

$$\psi = \beta [(1 - p_F)(\psi + d) + p_F(\psi + d_\kappa) u'((\psi + d_\kappa)t_{HF})], \quad (9)$$

with  $q_F = (\psi + d_\kappa)t_{HF}$ . If  $\psi = \psi^*$ , then any  $t_{HF} \geq \underline{q}(\psi^*)/(d/(1 - \beta) - \kappa)$  is optimal. In this case,  $q_F = \underline{q}(\psi^*)$ .

Case 3: Assume  $(p_F, \beta) \in R_3$ , or  $(p_F, \beta) \in R_2$  and  $\kappa/d \leq \tilde{\kappa}$ . Also, define  $\psi_c \equiv \beta(1 - p_F)(2d - \kappa)/[1 - 2\beta(1 - p_F)]$ . The following sub-cases arise: a) If  $\psi > \psi_c$ , then the optimal  $t_{HF}, t_{FF}, q_F$  are as in Case 1 above. b) If  $\psi < \psi_c$ , then the optimal  $t_{HF}, t_{FF}, q_F$  are as in Case 2 above. c) In the knife-edge case  $\psi = \psi_c$ ,  $t_{HF}, t_{FF} > 0$  and both (8) and (9) hold. The optimal choices  $t_{HF}, t_{FF}$  cannot be uniquely pinned down, but  $q_F$  is uniquely given by  $q_{F,c} \equiv \{q : u'(q) = (1 - p_F)/p_F\}$ .

*Proof.* See Appendix A. □

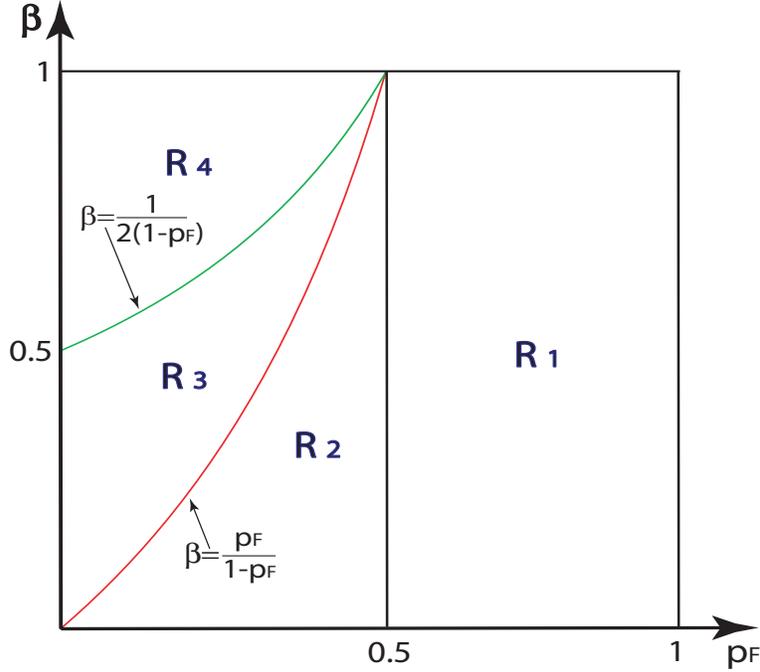


Figure 1: Parameter Values and Regions

Lemma 5 has an intuitive explanation. The term  $\kappa$  creates a “wedge” between the asset valuations of a seller and a buyer who come from different countries. The buyer knows that if she meets with a foreign seller, she will have higher purchasing power if she carries the foreign asset (which represents a home asset to the seller). On the other hand, meeting the seller is not guaranteed, and if the buyer stays unmatched in the foreign *DM*, she will receive the lower dividend associated with the foreign asset. A reverse story applies regarding the optimal choice of  $t_{HF}$ . If the buyer carries a large amount of the home asset and matches in the foreign *DM*, she will have lower purchasing power because, for every unit of  $t_{HF}$  that she passes to the seller, the latter will receive a lower dividend in the forthcoming *CM*. Not surprisingly, when  $p_F$  is high, the buyer decides to carry out trades in the foreign *DM* using the foreign asset. On the other hand, when it is less likely to match abroad (low  $p_F$ ), the buyer prefers to carry her home asset, even when trading in the foreign country. In intermediate cases, either regime can

arise, depending on the asset price, which affects the holding costs.<sup>11</sup>

The relationship between the optimal  $t_{HF}$  and  $t_{FF}$  with  $p_F$  is straightforward. What is perhaps more surprising is the fact that, in  $R_2$ , the buyer chooses to trade with the foreign asset when  $\kappa$  is relatively high. This might seem counterintuitive at first, given that a high  $\kappa$  means a low dividend for a buyer who did not dispose of the foreign asset. But one should not forget that a high  $\kappa$  also implies low purchasing power for the buyer, if she carries only  $t_{HF}$ . These forces have opposing effects. Whether it is optimal to trade with  $t_{HF}$  or  $t_{FF}$  depends on the value of  $\beta$ . When  $\kappa$  is high, the buyer realizes that she might receive a lower dividend, but this will happen *tomorrow*. On the other hand, when the buyer is trying to buy some  $q$  and pay with  $t_{HF}$ , the seller will receive a lower dividend in the current period's *CM*, i.e. *tonight*. Thus, a high  $\kappa$  dictates the use of  $t_{FF}$  in foreign meetings, as long as agents are not very patient. As an extreme case, consider points in  $R_2$  that are close to the origin, and suppose  $\kappa$  is very high. Although  $p_F$  is tiny, the buyer chooses to trade with  $t_{FF}$  because, in this region,  $\beta$  is also tiny.

So far we have analyzed the optimal behavior of agents for given (symmetric, steady state) asset prices. To complete the model, we incorporate the exogenous supply of assets in the analysis and treat  $\psi$  as an equilibrium object. For the reader's convenience, before we state the proposition that characterizes equilibrium, we repeat the definitions of some objects introduced above, and we define a few new objects.<sup>12</sup>

$$\begin{aligned}
q_{F,1}^* &\equiv \left\{ q : u'(q) = 1 + \frac{\kappa(1-\beta)(1-p_F)}{d} \right\} \\
q_{F,2}^* &\equiv \underline{q}(\psi^*) = \left\{ q : u'(q) = \frac{d}{d - (1-\beta)\kappa} \right\} \\
q_{H,c} &\equiv \left\{ q : u'(q) = \frac{d(1-2p_F) - \kappa(1-\beta)(1-p_F)}{p_H[d - \beta\kappa(1-p_F)]} \right\} \\
q_{F,c} &\equiv \left\{ q : u'(q) = \frac{1-p_F}{p_F} \right\} \\
T_1^* &\equiv \frac{(1-\beta)(q^* + q_{F,1}^*)}{d} \\
T_2^* &\equiv (1-\beta) \left[ \frac{q^*}{d} + \frac{\underline{q}(\psi^*)}{d - \kappa(1-\beta)} \right] \\
T_c &\equiv \frac{1-2\beta(1-p_F)}{d - \kappa\beta(1-p_F)} (q_{H,c} + q_{F,c}) < T_2^*
\end{aligned}$$

<sup>11</sup> An interesting point to notice is that in Case 3, for  $\psi$  in the neighborhood of  $\psi_c$ , the nature of the agent's optimal choice transforms fully. For  $\psi = \psi_c - \epsilon$ ,  $\epsilon > 0$ ,  $t_{FF} = 0$  and  $t_{HF} > 0$ . But for  $\psi = \psi_c + \epsilon$ ,  $t_{HF} = 0$  and the agent uses only foreign assets as a medium of exchange. Despite this dramatic change in the agent's portfolio composition, as  $\psi$  crosses the critical value  $\psi_c$ , the amount of special good purchased is continuous in  $\psi$ . To see this point, just set  $\psi = \psi_c + \epsilon$  in (8) and  $\psi = \psi_c - \epsilon$  in (9). It is easy to show that, as  $\epsilon \rightarrow 0$ ,  $q_F \rightarrow q_{F,c}$  in both cases.

<sup>12</sup> A comment on notation: when we write  $q_{F,1}^*$ , the asterisk refers to the fact that this is the highest value that  $q_F$  can reach and the number 1 refers to the case, in particular Case 1.

$$\psi_c \equiv \frac{\beta(1-p_F)(2d-\kappa)}{1-2\beta(1-p_F)}$$

$$\tilde{\kappa} \equiv \frac{(1-2p_F)}{(1-p_F)(1-\beta)}$$

**Proposition 1.** *For all parameter values, in equilibrium,  $t_{FH} = 0$ . Moreover:*

a) *If  $(p_F, \beta) \in R_1$ , or  $(p_F, \beta) \in R_2$  and  $\kappa/d \geq \tilde{\kappa}$ , equilibrium is characterized by “local asset dominance” and agents trade in the foreign DM using only foreign assets. Hence,  $t_{HF} = 0$ ,  $t_H = t_{HH}$ , and  $t_F = t_{FF}$ . If  $T \geq T_1^*$ , then  $\psi = \psi^*$ ,  $q_H = q^*$ ,  $q_F = q_{F,1}^* < q^*$ , and  $t_H = T - q_{F,1}^*(1-\beta)/d$ . If  $T < T_1^*$ , then  $\psi > \psi^*$ ,  $q_H < q^*$ , and  $q_F < q_{F,1}^*$ . For any  $T$  in this region,  $\partial\psi/\partial T < 0$ ,  $\partial q_H/\partial T > 0$ , and  $\partial q_F/\partial T > 0$ .*

b) *If  $(\beta, p_F) \in R_4$ , an “international asset” equilibrium arises and agents use their home assets in all DM’s, i.e.  $t_{FF} = 0$ , so  $t_F = 0$  and  $t_H = T$ . If  $T \geq T_2^*$ , then  $\psi = \psi^*$ ,  $q_H = q^*$ , and  $q_F = \underline{q}(\psi^*) < q^*$ . If  $T < T_2^*$ , then  $\psi > \psi^*$ ,  $q_H < q^*$ , and  $q_F < \underline{q}(\psi^*)$ . For any  $T$  in this region,  $\partial\psi/\partial T < 0$ ,  $\partial q_H/\partial T > 0$ , and  $\partial q_F/\partial T > 0$ .*

c) *If  $(\beta, p_F) \in R_3$ , or  $(\beta, p_F) \in R_2$  and  $\kappa/d < \tilde{\kappa}$ , a “mixed regime” equilibrium arises in the sense that either local asset dominance or international asset could arise depending on  $T$ . If  $T > T_c$ , we are in the international asset regime. For  $T$ ’s in this region,  $\psi \in [\psi^*, \psi_c)$ ,  $q_H \in (q_{H,c}, q^*]$ , and  $q_F \in (q_{F,c}, q_{F,2}^*]$ . If  $T < T_c$ , we switch to a local asset dominance equilibrium and  $\psi > \psi_c$ ,  $q_H < q_{H,c}$ , and  $q_F < q_{F,c}$ . In the knife-edge case where  $T = T_c$ , buyers purchase  $q_F = q_{F,c}$  in the foreign DM using any combination of home and foreign assets.*

*Proof.* See Appendix A. □

A unique steady state equilibrium exists for all parameter values. Buyers never carry foreign assets in order to trade in their home DM. Under parameter values summarized as Case 1 in Lemma 5, an equilibrium with local asset dominance arises. Buyers choose to trade in the foreign DM using the foreign asset only, because the positive effect (high purchasing power) of holding foreign assets dominates the negative effect (low dividend). There exists a critical level  $T_1^*$  that captures the *liquidity needs* of the economy. For  $T < T_1^*$ , increasing  $T$  helps buyers purchase more special good in both DM’s. Hence, the marginal valuation of one unit of the claim is higher than  $\psi^*$ , i.e. the price of the asset in a world where the Lucas-tree serves only as a store of value. The difference  $\psi - \psi^*$  represents a *liquidity premium* because it reflects a premium in the valuation of the asset that stems from its second role (as a medium of exchange). When  $T \geq T_1^*$ , the asset’s liquidity properties have been exploited (increasing  $T$  does not help buyers purchase more good) and  $\psi = \psi^*$ . When this is true, the cost of carrying the home asset is zero, so local buyers absorb all the excess supply,  $t_H = T - q_{F,1}^*(1-\beta)/d$ .

A similar analysis applies when parameter values are as in Case 2 in Lemma 5. We refer to this case as an international asset equilibrium because agents use their home asset as means

of payment everywhere in the world. The asset supply that captures the liquidity needs of the economy is given by  $T_2^*$ . When  $T \geq T_2^*$ , the asset price is down to its fundamental value and  $q_H, q_F$  have reached their upper bounds. Notice that in this case, buyers do not purchase any foreign assets in the  $CM$ , i.e.  $t_F = 0$  and  $t_H = T$ . However, this does not mean that no agent ever holds any foreign assets. Sellers from country  $i$  who got matched with foreign buyers get paid with, and therefore hold some, asset  $j$  (which represents a home asset to the buyers). This will be important in the next section where we will compute asset home bias.

When parameter values are as in Case 3 in Lemma 5, we can end up with local asset dominance or international asset equilibrium depending on  $T$ , which directly affects  $\psi$ . Equilibria with local asset dominance arise if and only if  $T > T_c$ . When  $T$  is large, equilibrium prices are relatively low, which means that the cost of carrying the asset is relatively low. When the cost of carrying assets is low, the term  $\kappa$  becomes relatively more important. Recall from the discussion of Lemma 5 that it is precisely when  $\kappa$  is relatively high that agents choose to use the foreign asset in order to trade in the foreign  $DM$  (this becomes obvious when  $(\beta, p_F) \in R_2$ ). On the other hand, if  $T$  is very small, the cost of carrying assets is large, so the term  $\kappa$  becomes less relevant. For intermediate values of  $(\beta, p_F)$  (remember we are in Case 3), this leads the buyers to optimally choose their home asset to trade in all  $DM$ 's.

## 5 Home Bias and High Turnover

In this section we use our model to revisit a well-known puzzle in the international finance literature: the coexistence of equity and consumption home bias with a higher turnover rate for foreign relative to domestic equities. To that end, we derive three predictions of our model and we show that they are in line with observations in the data. These predictions include: the existence of asset home bias, the positive correlation between asset and consumption home bias, and the higher turnover rate of foreign over domestic assets. We focus on the local asset dominance equilibrium, which is empirically relevant as it implies that countries constitute “currency areas”. Moreover, the quantitative exercise in Section 6 confirms that US data suggest parameter values that support this very equilibrium. For completeness, we treat the case of international asset equilibrium in the accompanying Online Appendix, and we show that all the theoretical results are preserved.

### 5.1 Preliminaries

In this section we establish some useful properties of the equilibrium values of  $q_H, q_F$  and  $t_H, t_F$ , which will lead us to our discussion of asset and consumption home bias as well as asset

turnover rates. For given  $\psi$ , the first-order conditions (7) and (8) implicitly define  $q_H, q_F$  as functions of the probabilities  $p_H, p_F$ . We have

$$\begin{aligned} q_H &= q_H(p_H) \equiv \left\{ q : u'(q) = 1 + \frac{\psi - \beta(\psi + d)}{\beta p_H(\psi + d)} \right\}, \\ q_F &= q_F(p_F) \equiv \left\{ q : u'(q) = \frac{\psi - \beta(\psi + d)(1 - p_F)}{\beta p_F(\psi + d)} \right\}. \end{aligned}$$

The produced quantities can be used to characterize the volumes of assets that change hands in bilateral meetings. From the bargaining solution, we have

$$\begin{aligned} t_H(p_H) &= \frac{q_H(p_H)}{\psi + d}, \\ t_F(p_F) &= \frac{q_F(p_F)}{\psi + d}. \end{aligned}$$

**Lemma 6.** *a) If  $T < T_1^*$ , the following results hold: i) The functions  $q_H(p_H), q_F(p_F)$  and  $t_H(p_H), t_F(p_F)$  are strictly increasing. ii) For every  $p_H = p_F = p \in (0, 1)$ ,  $q_H(p) > q_F(p)$ . iii) For any  $p_F \in (0, 1)$ , define  $\tilde{p}_H(p_F) \equiv \{p : q_H(p) = q_F(p_F)\}$ . Then,  $\tilde{p}_H(p_F) < p_F$ , and for any  $p_F \in (0, 1)$ ,  $p_H > \tilde{p}_H(p_F)$  implies  $q_H(p_H) > q_F(p_F)$ . iv) For any  $p_F \in (0, 1)$ ,  $p_H > \tilde{p}_H(p_F)$  implies  $t_H(p_H) > t_F(p_F)$ .*

*b) If  $T \geq T_1^*$ ,  $q_H = q^* > q_{F,1}^* = q_F$ . Also,  $t_H = T - q_{F,1}^*(1 - \beta)/d > q_{F,1}^*(1 - \beta)/d = t_F$ .*

*Proof.* See Appendix A. □

When the economy is liquidity constrained, i.e.  $T < T_1^*$ , the equilibrium quantity of special good purchased in a certain  $DM$ , and the volume of the traded asset, are increasing in the probability of visiting that  $DM$ . Since,  $q_H(p)$  is strictly larger than  $q_F(p)$ , for all  $p \in (0, 1)$ , agents purchase greater amounts of the home special good, even when the probability of trading in the local  $DM$  is smaller than the probability of trading in the foreign  $DM$  (but not much smaller). Moreover, since trade in a certain  $DM$  is facilitated by the local asset, the condition which guarantees that  $q_H > q_F$  (namely  $p_H > \tilde{p}_H(p_F)$ ), will also guarantee that  $t_H > t_F$ . On the other hand, if  $T \geq T_1^*$ , we have  $q_H > q_F$  and  $t_H > t_F$ , regardless of  $p_H, p_F$ .

## 5.2 Asset Home Bias

In this section, we derive sufficient conditions under which the model's predicted asset portfolio is biased toward domestic assets. The result is summarized in Proposition 2.

**Proposition 2.** *a) Assume that  $T < T_1^*$ . For any  $p_F \in (0, 1)$ , let  $p_H > \tilde{p}_H(p_F)$ . Then agents' portfolios exhibit home bias in the sense that the home asset share in the entire portfolio is greater than fifty percent.*

Formally, the home asset share is

$$HA = \frac{2t_H + p_F t_F}{2T} > 0.5. \quad (10)$$

b) Assume that  $T \geq T_1^*$ . Agents' portfolios exhibit home bias for any  $p_H, p_F$ . Formally,

$$HA = \frac{2 \left( T - \frac{1-\beta}{d} q_{F,1}^* \right) + p_F \frac{1-\beta}{d} q_{F,1}^*}{2T} > 0.5. \quad (11)$$

*Proof.* See Appendix A. □

The details of the derivation of the asset home bias formula can be found in Appendix B. To understand why the left-hand side of (10) represents the home asset share in the agents' portfolio, focus on the citizens of a certain country. In the denominator,  $T$  stands for the citizens' total asset holdings. It is multiplied by two because we account for asset holdings in both the *DM* and the *CM* (we assume equal weight). The numerator represents the home asset holdings. The term  $2t_H$  stands for the buyers' asset holdings in the *CM* and in the *DM*. The term  $p_F t_F$  is the amount of the home asset held by sellers who matched with foreign buyers. The formula in (11) admits a similar interpretation.

To understand the result, first consider the case in which  $T < T_1^*$ . Then, portfolios exhibit home bias as long as trading opportunities at home are not significantly less abundant than trading opportunities abroad. This follows directly from Lemma 6. Intuitively, when the economy is liquidity constrained, agents incur positive costs to hold the domestic and the foreign asset. Hence, they choose the optimal mix of assets in anticipation of their purchasing needs for special goods, which are governed by the trading opportunities. The higher the probability for a successful match in a market, the higher the demand for the asset that facilitates trade there.

When  $T \geq T_1^*$ , agents purchase the quantities  $q^*$  and  $q_{F,1}^*$ . The term  $q^*$  is independent of the matching probabilities, while  $q_{F,1}^*$  is rising in  $p_F$  but never exceeds  $q^*$ . Hence, for the purpose of acquiring special goods, the demand for foreign assets cannot exceed the demand for domestic assets. Moreover, as soon as the liquidity needs of the economy are satisfied, asset prices fall to fundamental values. Then, domestic assets become useful saving tools, while foreign assets remain costly to hold. Hence, agents hold the domestic asset both as a saving tool and in order to engage in trade at home, while they only acquire enough foreign assets to satisfy their purchasing needs abroad.

That agent portfolios are biased toward domestic assets is a well-known empirical regularity. In their seminal paper, French and Poterba (1991) document that, in 1989, the five largest economies held ninety percent of their wealth in domestic equities. Although home bias is not as severe in recent decades, it does persist in the data. To document this fact, we rely on "The External Wealth of Nations Mark II" database provided by Lane and Milesi-Ferretti (2007).

This database combines countries' national statistics with data from the International Monetary Fund (IMF), the World Bank, and the Bank of International Settlements. In the database, foreign portfolio equity assets for a country represent the value of all holdings of shares of firms headquartered abroad, where the stake does not exceed ten percent of the firm's value.<sup>13</sup> Foreign portfolio equity liabilities for a country represent the value of similar holdings of domestic firms by foreigners. The holdings for a given year represent positions as of December 31 of that year and the values are converted into US dollars using end-of-period exchange rates.

We combine these statistics with data on stock market capitalization from the World Development Indicators (WDI). We focus on twenty-seven OECD countries during the 1997-2007 period. To measure equity home bias, we employ a methodology similar to Collard, Dellas, Diba, and Stockman (2009). First, we compute international diversification as follows

$$\text{Int'l Dvrsf.} = \frac{\text{Foreign Portfolio Equity Assets}}{\text{Stock Mkt Cap} + \text{Foreign Portfolio Equity Assets} - \text{Foreign Portfolio Equity Liab.}}$$

Then, we capture equity home bias through the home equity share

$$\text{HE} = 1 - \text{Int'l Dvrsf.}$$

Table 1: Average Home Equity Share of Portfolio, 1997-2007

Country	HE	Country	HE	Country	HE
Australia	0.8374	Germany	0.5951	Norway	0.5493
Austria	0.4279	Greece	0.9536	Poland	0.9850
Belgium	0.5720	Hungary	0.9450	Portugal	0.7011
Canada	0.7432	Italy	0.6364	Spain	0.8589
Chile	0.8366	Japan	0.8972	Sweden	0.6290
Czech Republic	0.8382	Korea, Republic of	0.9538	Switzerland	0.5659
Denmark	0.6218	Mexico	0.9280	Turkey	0.9772
Finland	0.7206	Netherlands	0.4018	United Kingdom	0.6906
France	0.7523	New Zealand	0.6502	United States	0.8531
		<i>OECD Average</i>	<i>0.7452</i>		

We report the home equity shares for all twenty-seven OECD countries in Table 1. The typical developed country in our sample exhibits equity home bias, since average home equity shares over the period are in excess of the fifty-percent benchmark that characterizes a

<sup>13</sup>This is the standard definition of equity portfolio investment in the finance literature. More concentrated ownership is recorded as foreign direct investment (FDI). The finance literature assumes that the motivation behind such investments is different from typical portfolio investment.

fully-diversified portfolio in our model. The average domestic equity share among the OECD countries is 74 percent. Moreover, the corresponding statistic for the US is as high as 85 percent. Thus, it is reasonable to conclude that home bias is a stylized fact for developed economies in recent decades.

### 5.3 Consumption and Asset Home Bias

In this model, consumption and asset home bias coexist. Proposition 3 states the result.

**Proposition 3.** *Define  $C_F$ ,  $C_T$ , and  $C_H$  as the value of foreign (or imported) consumption, total consumption, and consumption produced at home, respectively.*

- a) *Assume  $T < T_1^*$ . For any  $p_F \in (0, 1)$ , let  $p_H > \tilde{p}_H(p_F)$ . Then  $C_H > C_F$ , implying  $\frac{C_H}{C_T} > 0.5$ .*
- b) *Assume  $T \geq T_1^*$ . For any  $p_H, p_F$ , we have  $\frac{C_H}{C_T} > 0.5$ .*

*Proof.* See Appendix B. □

The details of the accounting of consumption can be found in Appendix B. For each agent in the model, imported consumption consists of special goods bought from foreign sellers as well as fruit obtained from abroad when net claims to foreign trees are positive. On the contrary, domestic consumption includes special goods bought in domestic *DM* meetings as well as fruit obtained not only when net claims to the domestic tree are positive, but also via work.

Then, the results follow from Proposition 2 and Lemma 6. Suppose that the parameters are restricted such that the economy exhibits asset home bias. Since, at the country level, more domestic assets are held relative to foreign ones, net aggregate claims to foreign trees fall short of net claims to domestic trees. With respect to the *DM*, Lemma 6 ensures that the quantity (and value) exchanged (and consumed) in a domestic meeting exceeds the quantity (and value) exchanged in a foreign meeting. Thus, for any non-negative work effort, domestic consumption exceeds imported consumption. Finally, Lemma 6 and Proposition 3 suggest that agents buy more consumption goods from countries whose assets they hold in higher amounts. Thus, the model yields the following testable prediction: bilateral imports are positively correlated with bilateral asset positions.

The above discussion demonstrates that consumption and asset home bias are intimately related in the model. When the economy is liquidity constrained, asset and consumption home bias arise as long as trading opportunities at home are not significantly less abundant than trading opportunities abroad. The latter occurs if the probability with which agents visit the domestic relative to the foreign market is higher. It is reasonable to argue that such parameter restriction would hold if agents have a stronger preference for domestic goods, experience more frequent positive shocks to domestic consumption, or simply possess superior information over

the quality of domestic goods. These assumptions are standard in the home bias literature (see Heathcote and Perri (2007), Collard, Dellas, Diba, and Stockman (2009) and Hnatkowska (2010)). Moreover, in the quantitative analysis in Section 6, we use US data to verify that the parameter restrictions indeed hold.

In contrast, when the liquidity needs of the economy are satisfied, consumption and asset home bias coexist regardless of the relative trading opportunities in domestic and foreign markets. The key insight from this case is that domestic assets are useful saving tools, while foreign assets are not due to the friction captured by the term  $\kappa$ . Hence, agents hold the domestic asset both as a saving tool and in order to engage in trade at home, while they only acquire enough foreign assets to satisfy their purchasing needs abroad. As is apparent in the next subsection, this very mechanism lies at the heart of the model's ability to reconcile the coexistence of asset and consumption home bias with a higher turnover rate of foreign over domestic assets.

The model's prediction that consumption and asset home bias coexist is supported by a large empirical literature. For example, Heathcote and Perri (2007) and Collard, Dellas, Diba, and Stockman (2009) use recent data on developed countries to show that more open economies (in terms of trade as a fraction of GDP) exhibit lower equity home bias. Rather than revisiting the facts that these authors document, we opt to test the second prediction of our model that we derive above—namely, bilateral imports are positively correlated with bilateral asset positions.

To that end, we obtain *bilateral* portfolio equity holding data from the Coordinated Portfolio Investment Survey (CPIS) database provided by the World Bank. The same definition of portfolio equity asset for a country discussed in the previous subsection applies to this database. Indeed, Lane and Milesi-Ferretti (2007) utilize this database, among others, in their computations of total external asset and liability positions for each country. The advantage of the database is that it contains *bilateral* equity holdings for the set of OECD countries considered above during the second half of the studied decade, 2002-2007. The disadvantage is that a number of positive bilateral observations are not made publicly available for security reasons. Thus, domestic equity shares cannot be computed, since missing observations will bias home equity shares upward. Consequently, we use the database to simply test whether there exists a positive correlation between bilateral imports and bilateral equity asset positions (for country-pairs for which data are available).

To compute bilateral equity holding shares, we combine the CPIS data with stock market capitalization data from WDI. In a given year, we define an asset share for country  $i$  from country  $j$  to be the ratio between the value of country  $i$ 's holdings of shares of firms headquartered in country  $j$  and country  $i$ 's stock market capitalization. This statistic is the empirical counterpart to country  $i$ 's foreign asset share derived in the model—namely, the stock market capitalization for country  $i$  represents its total asset supply, while  $i$ 's asset position in  $j$  represents  $i$ 's demand for  $j$ -assets. We merge these statistics with annual bilateral imports of goods obtained from

Stats.Oecd.<sup>14</sup> To compute bilateral import shares, we use GDP data from Stats.Oecd. We define an import share for country  $i$  from country  $j$  to be the ratio between the value of country  $i$ 's imports from country  $j$  and country  $i$ 's GDP.

Table 2 shows the cross-country correlation between bilateral equity holding and import shares for the 2002-2007 period. The correlation ranges between 0.3581 and 0.4504 and it is highly statistically significant. Hence, it is reasonable to conclude that bilateral equity holding and import shares are positively correlated in the data. These results are supported by the existing literature. Aviat and Coeurdacier (2007) find that a 10% increase in bilateral trade raises bilateral asset holdings by 6% to 7% in a simultaneous gravity-equations framework.

Table 2: Bilateral Import and Equity Holding Shares, 2002-2007

year	correlation	p-value	# obs
2002	0.4504	0.0000	570
2003	0.3833	0.0000	584
2004	0.3581	0.0000	602
2005	0.3707	0.0000	593
2006	0.3898	0.0000	617
2007	0.3973	0.0000	614

Note: # obs = # available bilateral equity holding data points

## 5.4 Domestic and Foreign Asset Turnover Rates

The model's predictions on asset and consumption home bias discussed above are shared with the frameworks of Heathcote and Perri (2007), Collard, Dellas, Diba, and Stockman (2009), and Hnatkovska (2010). However, the additional prediction that relates domestic and foreign asset turnover rates discussed below is unique to the present model.

**Proposition 4.** *Define the turnover rates of home and foreign assets as*

$$TR_H = \frac{3p_H t_H + 2p_F t_F}{2t_H + p_F t_F}, \quad \forall T < T_1^*, \quad (12)$$

$$TR_H = \frac{\frac{1-\beta}{d} (3p_H q^* + 2p_F q_{F,1}^*)}{2 \left( T - \frac{1-\beta}{d} q_{F,1}^* \right) + \frac{1-\beta}{d} p_F q_{F,1}^*}, \quad \forall T \geq T_1^*, \quad (13)$$

<sup>14</sup> We use the Grand Total Import statistic, which includes imports across all commodity categories. Bilateral import data for the service sector are not available. In any case, including service trade should not affect our results, since services are mostly non-tradable and they account for a tiny portion of overall trade.

$$TR_F = \frac{2p_F}{2 - p_F}, \quad \forall T. \quad (14)$$

There exists a level of asset supply  $\tilde{T}$ , with  $T_1^* \leq \tilde{T} < \infty$ , such that  $T \geq \tilde{T}$  implies  $TR_F > TR_H$ .

Since the derivations of turnover rates are essential for the understanding of the Proposition, we present the proof in the main text.

*Proof of Proposition 4.* Following Warnock (2002), we define the foreign turnover rate as the ratio of the total volume of the foreign asset *traded* (sum of gross purchases and gross sales) by the citizens of the domestic country (numerator) over the total volume of the foreign asset *held* by the citizens of the domestic country (denominator).<sup>15</sup> For theoretical consistency, we define the domestic turnover rate as the ratio of the total volume of the domestic asset *traded* by the citizens of the domestic country over the total volume of the domestic asset *held* by the citizens of the domestic country. We continue to count asset holdings in both the *CM* and the *DM*, assuming equal weights for the two markets. It is understood that we count asset holdings at the end of each subperiod.

a) Suppose  $T < T_1^*$ .  $TR_F$  is given by (14). The numerator reflects all the trades of the foreign asset carried out by the citizens of the domestic country.<sup>16</sup> A measure  $p_F$  of buyers match in the foreign *DM* and exchange  $t_F$  units of the foreign asset to acquire the special good. They also purchase  $t_F$  units of the foreign asset in the *CM* in order to re-balance their portfolios. The denominator consists of the total holdings of the foreign asset, which amount to  $(2 - p_F)t_F$ . This expression includes the holdings of buyers in both the *CM* and the *DM*,  $2t_F$ , net of the amount given up by buyers of measure  $p_F$  who matched abroad. Since the term  $t_F$  appears in both the numerator and the denominator, it cancels out, yielding the constant turnover rate in (14).

$TR_H$  is given by (12). The numerator includes the amount of domestic assets that buyers buy,  $p_H t_H$ , and sellers sell,  $p_H t_H + p_F t_F$ , in order to re-balance their portfolios in the *CM* after successfully matching in the *DM*.<sup>17</sup> In addition, the numerator reflects the amount of domestic assets exchanged in the *DM* between sellers and domestic and foreign buyers, respectively,  $p_H t_H + p_F t_F$ . The denominator corresponds to total domestic assets held, which can be found in the numerator of the home-asset share expression in (10).

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<sup>15</sup> Warnock's (2002) definition uses values rather than volumes, but the two are identical in the steady state of our model. This is the standard definition of foreign turnover rate in the literature. For example, Tesar and Werner (1995) and Amadi and Bergin (2008) follow the same convention.

<sup>16</sup> To compute foreign turnover, we adopt the following accounting procedure: (i) In the *DM*, we count each transaction only once, since the asset flows across national borders only once and the transaction would therefore be recorded in the data only once; (ii) In the *CM*, we count both the amount of assets bought by the buyers and the amount sold by the sellers because the asset flows across national borders twice. This is consistent with the computation of foreign turnover in the data, where *gross* asset flows across borders are recorded.

<sup>17</sup> For theoretical consistency, we apply the convention from above when we compute domestic turnover rates—namely, we count transactions in the *DM* once and transactions in the *CM* twice. We discuss different methodologies applied by statisticians to compute domestic turnover in the data in the next subsection.

b) Suppose  $T \geq T_1^*$ . Clearly,  $TR_F$  is still given by (14), but  $TR_H$  differs. The logic of the calculations is identical. The main difference lies in the denominator. When  $T \geq T_1^*$ , not all domestic asset holdings are used as media of exchange in the  $DM$  because they have desirable saving properties. Following the same strategy as above and substituting asset holdings with the corresponding quantities of special goods exchanged in the  $DM$  obtains (13).  $TR_F$  is constant, so it is unaffected by the asset supply. However,  $TR_H$  is decreasing in  $T$ , for all  $T$ , and  $TR_H \rightarrow 0$  as  $T \rightarrow \infty$ . Therefore, there exists  $\tilde{T}$ , with  $T_1^* \leq \tilde{T} < \infty$ , such that  $T \geq \tilde{T}$  necessarily implies  $TR_F > TR_H$ .  $\square$

As in Propositions 2 and 3, there are two distinct cases to consider. If  $T < T_1^*$ , the domestic and foreign turnover rates are given by (12) and (14), respectively. However, unlike the home asset and consumption shares, the domestic turnover rate is *not* monotone in the trading probability at home. Hence, an unambiguous ranking between domestic and foreign turnover rates cannot be made in this case.

In contrast, when  $T$  exceeds  $T_1^*$ , agents purchase the quantities  $q^*$  and  $q_{F,1}^*$ , so increases in  $T$  have no effect on the quantities of special goods bought. Instead, increases in the asset supply raise the agents' domestic asset holdings, but have no effect on the amounts of domestic assets that change hands during a period, leading to a decrease in the turnover rate of the domestic asset. Intuitively, when the liquidity needs of the economy are satisfied, domestic assets are useful saving tools, while foreign assets are not due to the term  $\kappa$ . Hence, agents hold the domestic asset as a saving tool and use it to trade at home, while they only acquire enough foreign assets to satisfy their purchasing needs abroad. The domestic asset's desirable saving property results in a reduction in its turnover rate. On the other hand, agents offload foreign assets (to foreign sellers) at the first given opportunity, which leads to a relatively higher turnover rate of the foreign asset. Thus, the mechanism that drives the coexistence of asset and consumption bias also yields a higher turnover rate of foreign over domestic assets.

#### 5.4.1 Turnover Rates in the Data

In the theoretical analysis above, we adopted Warnock's (2002) definition of the foreign turnover rate, which is standard in the empirical literature. Given this definition, we constructed a theoretically-consistent definition of the domestic turnover rate. However, since the existing empirical literature has not considered the liquidity mechanism developed in this paper, it does not report measures of the theoretically-consistent domestic turnover rate. Instead, Warnock (2002) (and the literature in general) defines domestic turnover as the ratio of annual transactions on a market to its capitalization. The market for which estimates are reported is the stock exchange. However, as Amadi and Bergin (2008) point out, different stock exchanges in the US follow different conventions to compute turnover. In particular, the NASDAQ double counts

every transaction (both the sale and the purchase of a stock), while the remaining exchanges record the transaction only once. Consequently, the literature reports domestic turnover for the entire US stock exchange as well as for stock markets excluding the NASDAQ.

In order to relate to the empirical literature, we re-establish the turnover results using the empirically-relevant definition of domestic turnover. In the model, the  $CM$  represents the stock exchange. Its market capitalization is simply the total asset supply  $T$ . Following the double-counting (NASDAQ) convention, the transactions that constitute the numerator are all the trades (purchases and sales) of domestic claims by both domestic and foreign agents. Domestic agents who successfully matched in the domestic  $DM$  re-balance their portfolios as follows: a measure  $p_H$  of buyers buy  $t_H$  units, and measures  $p_H$  and  $p_F$  of sellers sell  $t_H$  and  $t_F$  units, respectively. In addition, a measure  $p_F$  of foreign buyers who matched in the domestic  $DM$  purchase  $t_F$  units of the domestic asset. Hence, the empirically-relevant turnover rate of home assets is

$$\begin{aligned} TR_H &= \frac{2(p_H t_H + p_F t_F)}{T}, & \forall T < T_1^*, \\ TR_H &= \frac{2^{\frac{1-\beta}{d}} (p_H q^* + p_F q_{F,1}^*)}{T}, & \forall T \geq T_1^*, \end{aligned} \quad (15)$$

where the second line is obtained by substituting out the asset holdings using the appropriate formulas for the liquidity-unconstrained case. Once again, the foreign turnover rate in (14) is constant, so it is unaffected by the asset supply. On the other hand, if  $T \geq T_1^*$ ,  $TR_H$  is decreasing in  $T$ , and  $TR_H \rightarrow 0$  as  $T \rightarrow \infty$ . Therefore, there exists  $\tilde{T}$ , with  $T_1^* \leq \tilde{T} < \infty$ , such that  $T \geq \tilde{T}$  implies  $TR_F > TR_H$ .

Naturally, the sufficient conditions derived above guarantee that foreign turnover exceeds domestic turnover computed following the single-counting convention (namely, excluding the NASDAQ). In that case the domestic turnover rate predicted by the model is exactly one half of expression (15) above.

Empirically, Tesar and Werner (1995) were the first to document considerably higher foreign over domestic turnover rates (as defined in this subsection) for the year 1989 for five major economies: United States, Japan, Germany, United Kingdom, and Canada. Warnock (2002) revised the authors' estimates for the United States and Canada and in addition presented new estimates for the pair of countries for the year 1997. The most up-to-date and comprehensive estimates of turnover rates are reported by Amadi and Bergin (2008). The authors repeat Warnock's (2002) exercise for the United States, Japan, Germany, and Canada during the 1993-2001 period employing two different datasets to estimate foreign equity positions, which constitute the denominator in the foreign turnover formula, and two different conventions to measure domestic turnover.

Amadi and Bergin (2008) report their findings in Tables 1 and 2 of their paper. Across all

measures and datasets, foreign turnover rates exceed domestic turnover rates. It is this coexistence of higher foreign over domestic turnover rates with consumption and asset home bias that puzzles the international finance literature. The magnitudes of the relative turnover rates, however, vary with the dataset and convention employed. Focusing on the estimates that rely on the International Financial Statistic (IFS) database provided by the IMF to measure foreign equity positions, and therefore foreign turnover, the mean relative foreign-to-domestic turnover rates during the 1993-2001 period for Canada, Germany, and Japan are 5.04, 2.45, and 2.26, respectively. The corresponding ratios for the US are 1.89 and 1.35, where the former excludes the turnover rate for NASDAQ from the domestic turnover measure.<sup>18</sup> Therefore, it is reasonable to conclude that foreign turnover rates are significantly higher than domestic turnover rates in a typical developed economy.

## 6 Quantitative Analysis

To complete the analysis, we evaluate the quantitative importance of the novel liquidity mechanism. In particular, we ask the following two questions: (i) Are the sufficient conditions on parameter values satisfied so that a typical developed economy exhibits consumption and asset home bias as well as a higher foreign over domestic turnover rate? (ii) How do the predicted asset home bias and foreign asset turnover rate compare to the observations in the data when the model's parameters are calibrated to match informative moments for a typical developed economy? In order to answer these questions, we engage in a calibration exercise using data on the US economy. In particular, we choose the key parameters in the model so as to match the import-to-GDP ratio and the domestic equity turnover rate in the US over the past decade. We then derive international asset portfolios and turnover rates of foreign assets predicted by the model, and we discuss how these moments compare to actual data.

### 6.1 Calibration

In order to proceed with the quantitative exercise, we choose functional forms that are common in the modern monetary literature. We follow Lagos and Wright (2005) and let the utility function in the decentralized market be  $u(q) = \log(q + b) - \log(b)$ , which ensures that  $u(0) = 0$ , as necessary in order to obtain a solution to the bargaining problem. Further, we

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<sup>18</sup>The authors repeat the computations using foreign equity positions obtained from the CPIS data for years 1997, 2001, and 2002. The mean relative foreign-to-domestic turnover rates across the three years for Canada, Germany, and Japan are 4.16, 1.65, and 1.89, respectively. The corresponding ratios for the US are 1.70 and 1.05, where the former excludes the turnover rate for NASDAQ from the domestic turnover measure.

let  $M(\mathcal{B}, \mathcal{S}) = \min\{\mathcal{B}, \mathcal{S}\}$ , which means that, in any  $DM$ , the shortest end of the market is always matched.

Next, we discuss a technical issue that arises in this adaptation of the Lagos and Wright (2005) model in a multi-country/international trade environment. Recall that all agents consume  $X^*$  in the  $CM$ . This parameter reflects the relative size of the  $CM$  compared to the  $DM$ .<sup>19</sup> Since, in our model, all the interesting economic activity takes place in the  $DM$ , we would like a relatively small  $X^*$ . However, if  $X^*$  is too small, agents who enter the  $CM$  with a lot of assets, for example sellers who just traded in the  $DM$ , might be induced to work negative hours.

To deal with this issue, in the quantitative exercise we assume that  $U(X) = X$ . Linear utility is a sub-case of the quasi-linearity assumed throughout the paper. Therefore, all the desirable properties of the value functions described earlier will continue to hold. Moreover, with linear utility, agents are indifferent between working many hours and consuming more, or working few hours and consuming less. Although there is a continuum of consumption and labor choices that are all optimal for the agent, different choices lead to different values of GDP, which is essential for accounting. We take advantage of this indeterminacy and assume that the agent always picks the pair of (optimal)  $X, H$  that involves the minimum amount of hours worked. This guarantees that the size of the  $CM$  sector is the minimum possible.<sup>20</sup>

With the functional forms in mind, we need to choose values for the following parameters  $(b, \beta, d, T, \xi, \sigma_H, \sigma_F, \kappa)$ . Notice a few points. First, as in any Lucas-tree model, the product  $dT$ , which represents the total dividend in a country, determines all equilibrium real variables. Thus, we need not take a stand on each one of these parameters individually. Consequently, we fix  $d = 1$ , and calibrate  $T$  below. Second, below we will be choosing a parameter value for  $\sigma_F$  in order to match the observed US import-to-GDP ratio. Since this statistic has been roughly fifteen percent over the past decade, it is reasonable to expect that, in any equilibrium,  $\sigma_F$  will never exceed  $\sigma_H$ . Thus, we normalize  $\sigma_H$  to unity and calibrate  $\sigma_F$  relative to  $\sigma_H$ . Third, recall that, in the quantitative analysis, we let  $M(\mathcal{B}, \mathcal{S}) = \min\{\mathcal{B}, \mathcal{S}\}$ . Moreover, we will let the measure of sellers  $\xi$  be 2. These two assumptions together imply that a buyer who gets to visit any  $DM$  will be matched with certainty. Ex ante, the probability with which a buyer gets to visit the foreign  $DM$  equals  $p_F = \sigma_F$ , which is one of the key parameters of our exercise.

Following Lagos and Wright (2005), we let  $b = 0.0001$ , which is positive, but small enough to minimize deviations from the log utility function. We let  $\beta = 0.97$ , since we will work with

<sup>19</sup> More precisely, a larger  $X^*$  implies that, for given asset holdings, agents will need to work more in the  $CM$  in order to be able to consume their desired amount of general good. Suppose we are interested in the percentage of total production that takes place in the  $CM$ . A higher  $X^*$  implies that the  $CM$  sector is relatively larger, since all the hours agents work in order to reach  $X^*$  contribute to the  $CM$  production.

<sup>20</sup> To see this point, consider an agent who enters the  $CM$  with assets whose value, in terms of the numéraire good, is  $y$ . The no-wealth-effect property of  $W$  implies that this agent will choose an amount of assets to carry into the next period's  $DM$  independently of the value of  $y$ . For simplicity, let the market value of assets carried into the next period be  $z$ . Because of linear utility, if  $y > z$ , the agent is equally happy between: (a) consuming  $y - z$  units of the  $CM$  good and working zero hours or (b) consuming  $y - z + \alpha$ , for any  $\alpha \in \mathbb{R}_+$ , and working  $\alpha$  hours. We assume that  $X = y - z$  and  $H = 0$ . If  $z > y$ , in a similar fashion we have  $X = 0$  and  $H = z - y$ .

annual data. Finally, we set  $\kappa$  to 0.0483, which represents a tax rate on foreign (relative to domestic) dividend income of 4.83%. We choose this value to match the additional tax rate on dividend income (over the US rate of 15%) that US investors face on average across the twenty-six OECD countries listed in Table 1 above.<sup>21</sup>

We are left with the following two parameters:  $\sigma_F$  and  $T$ . These two parameters are the most important ones in the model since they determine the type of equilibrium that arises.  $\sigma_F$  governs whether the equilibrium will be of the local-asset-dominance type, in which case local assets facilitate trade, or of the international-asset type, where agents rely on their respective domestic asset to trade everywhere. Moreover, conditional on being in a particular type of equilibrium, the value of  $T$  determines whether the economy is liquidity constrained or not, which further governs equilibrium prices and allocations. Consequently, we let the US data tell us what the values of these two parameters should be. In particular, we use the model's equations and the parameters discussed in previous paragraphs in order to find the unique values for  $\sigma_F$  and  $T$ , so that the model matches the following two moments in the US data: import-to-GDP ratio and domestic turnover rate.<sup>22</sup>

### 6.1.1 Benchmark Calibration

The first moment that we target in the calibration is the mean import-to-GDP ratio for the US over the 1997-2007 period, which amounts to 0.1475.<sup>23</sup> We derive imports and GDP for the model in detail in Appendix B. The second moment that we target is the average domestic turnover rate for the US stock market during the 1993-2001 period, computed excluding the NASDAQ. This is the literature's preferred estimate of domestic turnover. According to Table 1 in Amadi and Bergin (2008), the mean domestic turnover rate in the US was 0.72. The corresponding equation in the model is expression (15) divided by two.

Table 3 summarizes the benchmark calibration results. The calibrated model suggests a value for  $\sigma_F$  of 0.6819, and, as explained earlier, we also have  $p_F = \sigma_F = 0.6819$ . According to Lemma 5, this parameter value places the model's equilibrium in the local-asset-dominance region. Thus, domestic assets facilitate trade at home only, which is consistent with the fact that most countries in the world constitute independent "currency areas".

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<sup>21</sup>We obtain dividend income tax data for US investors in each foreign country from Deloitte for the year 2013. The foreign tax rates reflect tax treaties when applicable. The mean tax difference is 8.56% when tax treaties are not accounted for. Under such calibration, our model generates even higher home bias.

<sup>22</sup>In the model, all imports are consumed; therefore, imported consumption is equivalent to total imports. Moreover, GDP is straightforward to compute using the production approach. Thus, we follow Collard, Dellas, Diba, and Stockman (2009) and we approximate consumption home bias by the import-to-GDP ratio. In the data, imports include both consumption and intermediate goods. So, in order to directly measure consumption home bias, one would need to obtain disaggregate US import data and categorize goods into consumption and non-consumption goods. Then one would need to combine these data with total US consumption data in order to compute the share of consumption that is imported. We opt to use import-to-GDP ratios instead, purely in order to make our quantitative results comparable to the existing literature.

<sup>23</sup>Data are from WDI.

Table 3: Benchmark Calibration: Parameter Values and US Data Moments, 1993-2007

Parameter	Moment	Source
$N=2$	# countries in sample	-
$b=0.0001$	-	Lagos and Wright (2005)
$d=1$	normalization	-
$\sigma_H=1$	normalization	-
$\xi=2$	normalization	-
$\kappa=0.0483$	foreign dividend income tax diff. = 4.83%	Deloitte
$\beta=0.97$	annual real interest rate = 3%	-
$\sigma_F=0.6819$	import/GDP = 0.1475	WDI
$T=0.0701$	domestic turnover rate (excl. NASDAQ) = 0.72	Amadi and Bergin (2008)

Given that we are in Case 1 of Lemma 5, the properties of the equilibrium of the model are summarized under part *a*) in Proposition 1. In particular, if asset supply is above the critical level  $T_1^*$ , the economy is not liquidity constrained. This means that the asset price falls to the fundamental value and quantities reach their upper bounds. The quantity exchanged in a local meeting is given by  $q^*$  and the quantity exchanged in a foreign meeting is even lower and equals  $q_{F,1}^*$ , both of which are described in Definition 2 above. Using the parameter values discussed above, the critical asset supply value equals 0.06. The calibrated model suggests that the level of asset supply necessary in order to match the domestic turnover rate in the US is 0.0701.

Then, we can compute the equity portfolio and the foreign equity turnover rate predicted by the model in the unconstrained, local-asset-dominance equilibrium. Maintaining the notation corresponding to this type of equilibrium, the domestic equity share of a country's total portfolio is given by

$$HE_1^* = \frac{2T + \frac{1-\beta}{R}(p_F - 2)q_{F,1}^*}{2T}.$$

Applying this formula, the calibrated model yields a US home equity share of seventy-two percent. The corresponding statistic for the US data over the 1997-2007 period is eighty-five percent. Thus, the liquidity channel alone can account for a large portion of the observed equity home bias in the US data.

Furthermore, recall that the foreign asset turnover rate for this type of equilibrium is  $2p_F/(2-p_F)$ . Using the calibrated parameters, the model suggests that the foreign asset turnover rate is 1.03. The corresponding statistic in the US data is 1.36. Thus, the model generates a substantial foreign turnover rate that compares favorably to the US data. In addition, the predicted foreign turnover rate is 1.44 times higher than the domestic turnover rate. The corresponding ratio in the US data amounts to 1.89 (or 1.70 in the shorter CPIS sample). Hence, the model accounts for a large part of the turnover rate differentials in the US data. The intuition for this result lies in

the liquidity mechanism introduced in this paper. In equilibrium, domestic assets are held both for the purpose of future consumption and trade in domestic decentralized markets. Foreign assets, on the other hand, yield lower rates of return and hence they do not represent a useful saving technology. Thus, buyers acquire them purely for the purpose of trading abroad in the future and, if unlucky in the matching process, they use the first opportunity to liquidate them in the upcoming centralized market.

### 6.1.2 Robustness

In this section, we test the model’s ability to account for the differences in turnover rates using alternative measures of domestic turnover. To that end, we replace the second target above with the average domestic turnover rate for the US stock market during the 1993-2001 period, computed including the NASDAQ. According to Table 1 in Amadi and Bergin (2008), the mean alternative domestic turnover rate in the US was 1.13. This statistic combines turnover for the NASDAQ with the remaining stock exchanges in the US. Recall that the first turnover rate is computed using the double-counting methodology, while the second follows the single-counting methodology for transactions. In the model, the  $CM$  represents the entire stock exchange, so we cannot simultaneously apply the two different methodologies. We opt for the most conservative calibration, which applies the double-counting methodology to the entire stock exchange. Hence, the equation for domestic turnover in the model is expression (15).

Table 4: Robustness: Parameter Values and US Data Moments, 1993-2007

Parameter	Moment	Source
$N=2$	# countries in sample	-
$b=0.0001$	-	Lagos and Wright (2005)
$d=1$	normalization	-
$\sigma_H=1$	normalization	-
$\xi=2$	normalization	-
$\kappa=0.0483$	foreign dividend income tax diff. = 4.83%	Deloitte
$\beta=0.97$	annual real interest rate = 3%	-
$\sigma_F=0.6819$	import/GDP = 0.1475	WDI
$T=0.0893$	domestic turnover rate (incl. NASDAQ) = 1.13	Amadi and Bergin (2008)

Table 4 summarizes the calibration results from the robustness exercise. The resulting value of the asset supply,  $T$ , once again suggests that the economy is not liquidity constrained. The resulting equity home share for the US rises to seventy-eight percent, since the US asset base is now larger. Furthermore, the foreign turnover rate, which is uniquely pinned down by  $\sigma_F$ , remains at a high level of 1.03. However, the foreign-to-domestic turnover rate ratio falls to 0.92. Although the value falls short of the desired ratio of at least unity, the statistic compares

well to the observation in the data which amounts to 1.35 (or 1.05 in the shorter CPIS sample).

Overall, the model accounts for a large portion of the observed equity home bias in the US data in the two exercises. Admittedly, the ratio of foreign to domestic turnover falls short of the one observed in the US data, especially in the exercise that accounts for the NASDAQ. However, the model makes a successful step toward reconciling the behavior observed in the data, as it not only predicts high values of foreign turnover, but it is also the first to successfully generate a higher foreign relative to domestic turnover rate, while maintaining a strong quantitative link between consumption and asset home bias.

## 7 Discussion

### 7.1 Theory

In Sections 5 and 6, we documented three stylized facts, and we demonstrated that the model proposed in this paper can reconcile them qualitatively and quantitatively. However, in order to remain tractable, the model assumed away the risky nature of equity returns. We argue that incorporating stochastic asset returns into the model will not affect the qualitative results.

In order to feature a risk-sharing motive, a model needs to incorporate some curvature in agents' preferences. Given the assumed quasi-linear utility in the CM in our model, the uncertainty has to be revealed before the DM opens, as shown in Lagos (2010) (this point is also clarified in Section 11.3 of Nosal and Rocheteau (2011)). If our model features stochastic dividends, uncertainty will still affect agents' portfolios through the assets' property to facilitate exchange in the DM, i.e. through the liquidity channel. Consider the simple case in which the dividend processes for the two countries are *i.i.d.* over time, but negatively correlated in the cross section. Then, agents still hold the foreign asset only for the purpose of trading in the foreign DM.<sup>24</sup> Hence, the spirit of Proposition 2 is still valid: as long as trading opportunities abroad are not much more frequent than trading opportunities at home, the model will exhibit asset home bias. A similar logic applies to the results that concern consumption home bias, given the tight link between asset and consumption home bias in our framework. Moreover, agents will continue to offload the foreign asset at their earliest opportunity because, in expectation, the foreign asset remains a worse store of value. Hence, the liquidity channel remains the key mechanism that allows the model to reconcile the three stylized facts concurrently.

Moreover, our analysis is robust to the introduction of fiat money in the economy. In that case, agents will hold assets as a store of value due to positive asset returns since fiat money

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<sup>24</sup> Notice, however, that the relative cost of holding the foreign over the domestic asset is lower than in the benchmark deterministic economy due to the possibility that now the domestic asset may deliver a poor yield and therefore be a worse trade facilitator.

has negative or zero return, depending on the rate of inflation.<sup>25</sup> Suppose that we impose the standard currency-area assumption in the model; namely, trade in  $DM_i$  necessitates the use of  $i$ 's currency. If it is costly to convert foreign assets into home currency, due to a spread in the exchange rate, agents will hold  $i$ 's asset in order to trade in  $i$ 's currency. Hence, all of our results would be preserved. Moreover, since fiat money will serve as a medium of exchange in the economy, the model would predict that total domestic assets—the sum of real assets and fiat money—will turn over faster than total foreign assets. This is likely the case in the US, where the velocity of money is high. We abstract from fiat currency considerations because the valuation of fiat money is dependent on policy, which is beyond the scope of our paper.

In sum, the theoretical results of this paper are robust to several extensions of the model.

## 7.2 Data

In the quantitative application of this paper, we focus on equities. This asset class has been the focus of the existing empirical literature, and we utilize the same datasets that were employed in these papers. Moreover, the puzzle that constitutes the coexistence of home bias in assets and consumption with a higher foreign-relative-to-domestic asset turnover rate has been documented for equities only. One reason behind this is that data coverage for equities is by far the largest in the cross section of countries and the time series relative to other capital markets. For example, to compute the domestic turnover rate, one needs flows of assets among agents on the domestic market, relative to a stock base. The computation is straightforward for equities because they are traded on centralized exchanges, so data on flows within national borders are available. This is not the case for corporate bonds, which are typically traded in over the counter markets.

Furthermore, when linking the model to the data, we need to take a stand regarding the characteristics of the agents who are holding/trading the equities. We recognize that there are many market participants who hold and trade equities. According to Naess-Schmidt, Rytter, Hansen, and Jensen (2012), in a typical developed economy such as the European Union, individuals constitute 23% of cross-border portfolio investors, which refers to investments with ownership of less than 10%, while mutual funds (CIV's) account for 31%.<sup>26</sup> Hence, over half of cross-border investments are made by or on the behalf of private agents. The remainder include banks, pension and insurance companies, and governments, all of which arguably face different regulations/objective functions from individuals.

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<sup>25</sup> This idea is formalized in Geromichalos and Herrenbrueck (2012). The authors show that, although assets do not serve directly as means of payment, they may still carry positive liquidity premia, because agents wish to hold them as a hedge against inflation.

<sup>26</sup> The report was prepared for the European Commission and is available at the following webpage: [http://ec.europa.eu/taxation\\_customs/resources/documents/common/consultations/tax/venture\\_capital/tax\\_crossborder-dividend-paym.pdf](http://ec.europa.eu/taxation_customs/resources/documents/common/consultations/tax/venture_capital/tax_crossborder-dividend-paym.pdf).

We focus on individual investors and mutual funds that invest on their behalf. We assume that there are no agency problems and that mutual funds hold portfolios that maximize investors' objective functions. Thus, our model potentially speaks to roughly one half of the equity holdings. An issue may arise with respect to equity flows however. In the model, agents rebalance their portfolios for liquidity purposes. When agents' portfolios are managed by mutual funds, however, if individuals choose to rebalance their portfolios, the mutual fund may carry out the operation within the boundaries of the firm, in which case the flows would not be recorded in the data. This means that turnover rates would be underestimated in the data. Conversely, if mutual funds follow active investment policies (chasing short-run returns), then they likely rebalance asset portfolios more frequently than do individual agents for liquidity purposes. In this case, the turnover rates would be overestimated in the data.

Unfortunately, data on equity flows within or across countries that are differentiated by assets' country of origin and/or type of institution/agent do not exist. So, we are unable to separately quantify these different channels. Instead, we simply take the existing literature's observation that foreign equities turn over faster than domestic ones at face value, and we evaluate the calibrated model's ability to quantitatively account for the fact. Therefore, our quantitative results should be interpreted as quantifying the role that the liquidity channel alone plays in accounting for the observed facts.

## 8 Conclusion

In this paper, we study optimal asset portfolio choice in a two-country search-theoretic model of monetary exchange. We allow assets to not only represent claims on future consumption, but to also serve as media of exchange. In the model, trading in a certain country involves the exchange of locally produced goods for a portfolio of assets. Assuming that the net dividend from foreign assets is lower due to a friction (such as a cross-border dividend income tax), we provide sufficient conditions for the existence of two types of exchange regimes: local asset dominance and international asset equilibrium.

The model predicts that agents' portfolios exhibit home bias as long as trading opportunities abroad are not much more frequent than at home. Since agents hold larger amounts of domestic over foreign assets, they have larger claims to domestic over foreign consumption goods. Hence, the mechanism positively links asset and consumption home bias. More importantly, under additional parameter restrictions, foreign assets turn over faster than home assets because, while the former have desirable liquidity properties, they yield lower future consumption and are undesirable savings tools. Therefore, our theory offers an answer to a long-standing puzzle in international finance: a positive relationship between consumption and asset home bias coupled with higher turnover rates of foreign over domestic assets.

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## References

- AMADI, A. A., AND P. R. BERGIN (2008): “Understanding international portfolio diversification and turnover rates,” *Journal of International Financial Markets, Institutions and Money*, 18(2), 191–206.
- AVIAT, A., AND N. COEURDACIER (2007): “The geography of trade in goods and asset holdings,” *Journal of International Economics*, 71(1), 22–51.
- CAMERA, G., AND J. WINKLER (2003): “International monetary trade and the law of one price,” *Journal of Monetary Economics*, 50(7), 1531 – 1553.
- COLLARD, F., H. DELLAS, B. DIBA, AND A. STOCKMAN (2009): “Goods Trade and International Equity Portfolios,” *The University of Adelaide School of Economics Working Paper*, (14).
- DUFFIE, D., N. GARLEANU, AND L. H. PEDERSEN (2005): “Over-the-Counter Markets,” *Econometrica*, 73(6), 1815–1847.
- FRENCH, K. R., AND J. M. POTERBA (1991): “Investor Diversification and International Equity Markets,” *American Economic Review*, 81(2), 222–26.
- GEROMICHALOS, A., AND L. HERRENBRUECK (2012): “Monetary Policy, Asset Prices, and Liquidity in Over-the-Counter Markets,” Discussion paper.
- GEROMICHALOS, A., J. M. LICARI, AND J. SUAREZ-LLEDO (2007): “Monetary Policy and Asset Prices,” *Review of Economic Dynamics*, 10(4), 761–779.

- HEAD, A., AND S. SHI (2003): "A fundamental theory of exchange rates and direct currency trades," *Journal of Monetary Economics*, 50(7), 1555 – 1591.
- HEATHCOTE, J., AND F. PERRI (2007): "The International Diversification Puzzle Is Not As Bad As You Think," Working Paper 13483, National Bureau of Economic Research.
- HNATKOVSKA, V. (2010): "Home bias and high turnover: Dynamic portfolio choice with incomplete markets," *Journal of International Economics*, 80(1), 113–128.
- HOLMSTRÖM, B., AND J. TIROLE (1998): "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106(1), pp. 1–40.
- KIYOTAKI, N., AND J. MOORE (1997): "Credit Cycles," *Journal of Political Economy*, 105(2), 211–48.
- KOCHERLAKOTA, N. R. (1998): "Money Is Memory," *Journal of Economic Theory*, 81(2), 232–251.
- LAGOS, R. (2010): "Asset prices and liquidity in an exchange economy," *Journal of Monetary Economics*, 57(8), 913–930.
- (2011): "Asset Prices, Liquidity, and Monetary Policy in an Exchange Economy," *Journal of Money, Credit and Banking*, 43, 521–552.
- LAGOS, R., AND G. ROCHETEAU (2008): "Money and capital as competing media of exchange," *Journal of Economic Theory*, 142(1), 247–258.
- LAGOS, R., AND R. WRIGHT (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113(3), 463–484.
- LANE, P. R., AND G. M. MILESI-FERRETTI (2007): "The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970-2004," *Journal of International Economics*, 73(2), 223–250.
- LESTER, B., A. POSTLEWAITE, AND R. WRIGHT (2012): "Information, Liquidity, Asset Prices, and Monetary Policy," *Review of Economic Studies*, forthcoming.
- LEWIS, K. K. (1999): "Trying to Explain Home Bias in Equities and Consumption," *Journal of Economic Literature*, 37(2), 571–608.
- LUCAS, ROBERT E, J. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, 46(6), 1429–45.
- MATSUYAMA, K., N. KIYOTAKI, AND A. MATSUI (1993): "Toward a Theory of International Currency," *Review of Economic Studies*, 60(2), 283–307.

- MICHAELIDES, A. (2003): “International portfolio choice, liquidity constraints and the home equity bias puzzle,” *Journal of Economic Dynamics and Control*, 28(3), 555–594.
- NAESS-SCHMIDT, S., E. RYTTER, M. B. HANSEN, AND H. N. JENSEN (2012): “Taxation of cross-border dividend payments within the EU,” *Copenhagen Economics, report*.
- NOSAL, E., AND G. ROCHETEAU (2011): *Money, payments, and liquidity*. The MIT Press.
- ROCHETEAU, G., AND R. WRIGHT (2005): “Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium,” *Econometrica*, 73(1), 175–202.
- TESAR, L. L., AND I. M. WERNER (1995): “Home bias and high turnover,” *Journal of International Money and Finance*, 14(4), 467–492.
- WARNOCK, F. E. (2002): “Home bias and high turnover reconsidered,” *Journal of International Money and Finance*, 21(6), 795 – 805.
- WRIGHT, R., AND A. TREJOS (2001): “International Currency,” *The B.E. Journal of Macroeconomics*, 0(1).

## A Theory Appendix

*Proof of Lemma 2.* The term  $\kappa$  creates a wedge in the valuation of asset  $i$  between the seller (citizen of  $i$ ) and the buyer (citizen of  $j$ ). A positive amount of asset  $j$  will change hands only if all units of asset  $i$  have already been traded. Hence, the solution to the bargaining problem follows directly from answering the following two questions: 1) If the buyer carried unlimited amounts of asset  $i$  (which is foreign to him but local to the seller), what would be the optimal level of  $q$  to be traded? 2) If the buyer carried zero units of asset  $i$ , what would be the optimal level of  $q$  to be traded? It can be easily verified that the answer to the first question is given by  $\bar{q}(\psi) \equiv \{q : u'(q) = (\psi + d_\kappa)/(\psi + d)\}$ , which is clearly larger than  $q^*$ . Similarly, the answer to the second question is given by  $\underline{q}(\psi) \equiv \{q : u'(q) = (\psi + d)/(\psi + d_\kappa)\} < q^*$ .

Given these observations, the suggested solution follows naturally. When  $t_{ii} \geq \underline{q}(\psi_j)/(\psi_i + d)$ , asset  $j$  never changes hands (cases a and b). If  $t_{ii} \geq \bar{q}(\psi_j)/(\psi_i + d)q_i$  (case a),  $q_i$  is equal to the optimum. However, if  $t_{ii} \in [\underline{q}(\psi_j)/(\psi_i + d), \bar{q}(\psi_j)/(\psi_i + d)]$  (case b),  $q_i$  is bounded by the real value of the buyer’s asset- $i$  holdings, i.e.  $t_{ii}(\psi_i + d)$ . If  $t_{ii} < \underline{q}(\psi_j)/(\psi_i + d)$  (case c), some asset  $j$  is traded as well. If the total asset balances are such that  $\pi_i \geq \underline{q}(\psi_j)$  (case c1), then the buyer gives up all her asset  $i$  and enough asset  $j$  so that the quantity  $\underline{q}(\psi_j)$  can be purchased. On the other hand, if  $\pi_i < \underline{q}(\psi_j)$  (case c2), the buyer gives up all her asset holdings and purchases  $q_i = \pi_i = (\psi_i + d)t_{ii} + (\psi_j + d_\kappa)t_{ji}$ .  $\square$

*Proof of Lemma 4.* Focus on the symmetric, steady-state version of (5). Pick any  $(t_{HH}^0, t_{FH}^0)$ , with  $t_{FH}^0 > 0$  and define  $q^0 \equiv \{q : (\psi + d)t_{HH}^0 + (\psi + d_\kappa)t_{FH}^0\}$ , i.e. the quantity of the special good that  $(t_{HH}^0, t_{FH}^0)$  can buy. The buyer can purchase  $q^0$  by setting  $t_{FH} = 0$  and increasing her domestic asset holdings to  $t_{HH}^1 = q^0/(\psi + d)$ . We claim that the buyer always achieves a higher value if she purchases (the arbitrarily chosen)  $q^0$  with asset holdings  $(t_{HH}^1, 0)$  rather than  $(t_{HH}^0, t_{FH}^0)$ . To see this point notice that

$$\begin{aligned} V_H^B(t_{HH}^1, 0) &= [-\psi + \beta(\psi + d)]t_{HH}^1 + \beta p_H[u(q^0) - q^0], \\ V_H^B(t_{HH}^0, t_{FH}^0) &= [-\psi + \beta(\psi + d)]t_{HH}^0 + [-\psi + \beta(\psi + d_\kappa)]t_{FH}^0 + \beta p_H[u(q^0) - q^0]. \end{aligned}$$

After some algebra, one can show that  $V_H^B(t_{HH}^1, 0) > V_H^B(t_{HH}^0, t_{FH}^0)$  will be true if and only if  $t_{HH}^1 < t_{HH}^0 + t_{FH}^0$ . Multiply the last expression by  $\psi + d$  and add and subtract the term  $\kappa t_{FH}^0$  on the right-hand side. Then, one can conclude that  $V_H^B(t_{HH}^1, 0) > V_H^B(t_{HH}^0, t_{FH}^0)$  is true if and only if  $\kappa t_{FH}^0 > 0$ , which is true by assumption. Thus, the optimal strategy for the buyer is to purchase any desired  $q$  in the local  $DM$  by carrying only the domestic asset.

When  $\psi = \psi^*$ , the cost of carrying home assets is zero. We have  $t_{HH} \geq q^*/(\psi + d)$ , because the buyer carries any  $t_{HH}$  which is greater than the amount that buys her  $q^*$ .  $\square$

*Proof of Lemma 5.* The description of the optimal choice of  $t_{HF}$  is more complex than the one of  $t_{FF}$  for the following reason. If the buyer does not use foreign assets as media of exchange, we know that  $t_{FF} = 0$ . This is true because the cost of holding foreign assets is always strictly positive. However, with home assets, if  $\psi = \psi^*$ , the buyer might want to hold some home assets even if she is not using them to carry out transactions in the  $DM$ . To avoid these complications, first we focus on the case where  $\psi > \psi^*$ . Once we have established which assets serve as means of payments in the various cases, we let  $\psi = \psi^*$  and conclude the description of the optimal choice of home assets.

The symmetric, steady-state version of the buyer's objective function, i.e. equation (6), is

$$\begin{aligned} V_F^B(t_{HF}, t_{FF}) &= [-\psi + \beta(\psi + d)]t_{HF} + [-\psi + \beta(\psi + d_\kappa)]t_{FF} \\ &+ \beta p_F [u(q_F(t_{HF}, t_{FF})) - (\psi + d)x_H(t_{HF}, t_{FF}) - (\psi + d_\kappa)x_F(t_{HF}, t_{FF})], \quad (\text{a.1}) \end{aligned}$$

where  $x_H$  denotes the amount of domestic assets (with respect to the buyer's citizenship) and  $x_F$  the amount of foreign assets that change hands in a  $DM$  meeting in the foreign country. Since different asset holdings  $t_{HF}, t_{FF}$  lead to different expressions for the terms  $q_F, x_H$  and  $x_F$  (determined in Lemma 2), it is of no use to take first-order conditions in (a.1). Instead, we look into different combinations of  $t_{HF}, t_{FF}$  holdings. We start by ruling out several regions of asset holdings as strictly dominated. This allows us to narrow down the set of possibilities and eventually take first-order conditions in the remaining relevant regions. Figure 2 depicts the

regions described in this proof. The proof proceeds in several steps.

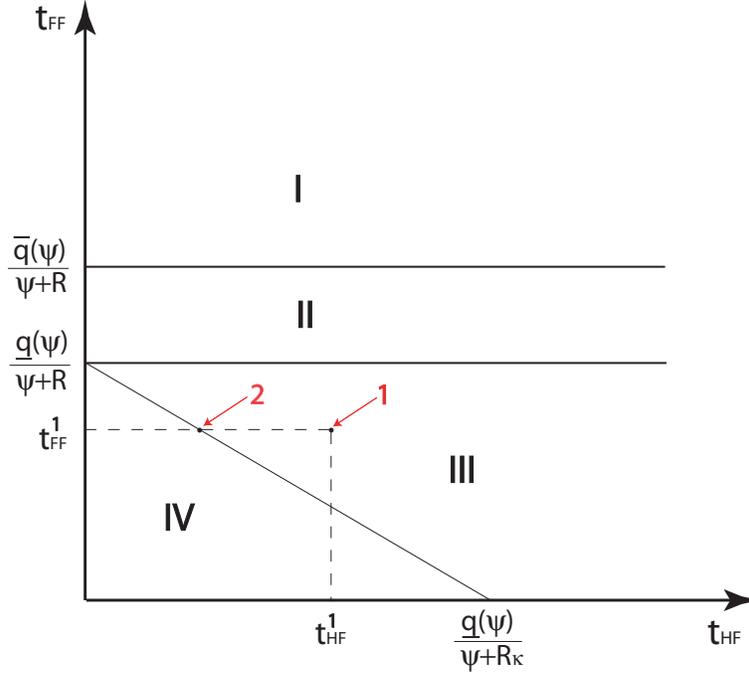


Figure 2: Regions

*Step 1:* The optimal  $t_{HF}, t_{FF}$  can never be such that  $t_{FF} > \bar{q}(\psi)/(\psi + d)$ . This represents region I in Figure 2 and case (a) in Lemma 2. To see why this claim is true, just notice that for any  $t_{HF}, t_{FF}$  in this region the buyer is already buying the highest possible quantity of special good, namely  $\bar{q}(\psi)$ . Hence, increasing  $t_{FF}$  has a strictly positive cost and no benefit.

*Step 2:* The optimal  $t_{HF}, t_{FF}$  can never be such that  $t_{FF} \in (\underline{q}(\psi)/(\psi + d), \bar{q}(\psi)/(\psi + d)]$  and  $t_{HF} > 0$ . This represents region II, excluding the vertical axis, in Figure 2 and case (b) in Lemma 2. To see why the claim is true, recall from Lemma 2 that for this region of asset holdings  $x_H = 0$ . Therefore, there is no benefit from carrying the home asset (but there is a cost). Hence, in this region the objective function becomes

$$V_F^B(t_{HF}, t_{FF}) = -\beta\kappa t_{FF} + \beta p_F [u((\psi + d)t_{FF}) - (\psi + d_\kappa)t_{FF}]. \quad (\text{a.2})$$

The optimal  $t_{FF}$  is determined by taking the first-order condition in (a.2).

*Step 3:* The optimal  $t_{HF}, t_{FF}$  cannot lie in the interior of region III in Figure 2 (and case (c1) in Lemma 2). To see why this is true, consider any point in the interior of region III, such as point 1 with coordinates  $(t_{HF}^1, t_{FF}^1)$ . The objective function for the buyer, evaluated at point 1, is

$$\begin{aligned} V_F^B(t_{HF}^1, t_{FF}^1) &= [-\psi + \beta(\psi + d)]t_{HF}^1 + [-\psi + \beta(\psi + d_\kappa)]t_{FF}^1 \\ &+ \beta p_F \left[ u(\underline{q}(\psi)) - (\psi + d) \left( \frac{\underline{q}(\psi) - (\psi + d)}{\psi + d_\kappa} \right) - (\psi + d_\kappa)t_{FF}^1 \right]. \end{aligned} \quad (\text{a.3})$$

Now consider the following experiment. Leave  $t_{FF}$  unchanged, but reduce  $t_{HF}$  so that the buyer can still purchase  $\underline{q}(\psi)$ . This is represented by point 2 in the Figure, with coordinates  $(t_{HF}^2, t_{FF}^2) = ((\underline{q}(\psi) - (\psi + d)t_{FF}^1)/(\psi + d_\kappa), t_{FF}^1)$ . The objective function evaluated at point 2 is

$$\begin{aligned} V_F^{\mathcal{B}}(t_{HF}^2, t_{FF}^2) &= [-\psi + \beta(\psi + d)] \frac{\underline{q}(\psi) - (\psi + d)t_{FF}^1}{\psi + d_\kappa} + [-\psi + \beta(\psi + d_\kappa)] t_{FF}^1 \\ &+ \beta p_F \left[ u(\underline{q}(\psi)) - (\psi + d) \left( \frac{\underline{q}(\psi) - (\psi + d)}{\psi + d_\kappa} \right) - (\psi + d_\kappa) t_{FF}^1 \right]. \end{aligned} \quad (\text{a.4})$$

It follows from (a.3) and (a.4) that

$$V_F^{\mathcal{B}}(t_{HF}^2, t_{FF}^2) - V_F^{\mathcal{B}}(t_{HF}^1, t_{FF}^1) = [-\psi + \beta(\psi + d)] (t_{HF}^2 - t_{HF}^1),$$

which is strictly positive by Lemma 3 and the fact that  $t_{HF}^2 < t_{HF}^1$ . Hence, the agent will never choose a point  $(t_{HF}, t_{FF})$  in the interior of region III.

*Step 4:* Finally consider the region of  $t_{HF}, t_{FF}$  represented by region IV (case (c2) of Lemma 2). The objective function in this region becomes

$$\begin{aligned} V_F^{\mathcal{B}}(t_{HF}, t_{FF}) &= [-\psi + \beta(\psi + d)(1 - p_F)] t_{HF} + [-\psi + \beta(\psi + d_\kappa)(1 - p_F)] t_{FF} \\ &+ \beta p_F u((\psi + d_\kappa)t_{HF} + (\psi + d)t_{FF}) \\ &= -a_1 t_{HF} - a_2 t_{FF} + a_3 u(a_4 t_{HF} + a_5 t_{FF}), \end{aligned} \quad (\text{a.5})$$

where we defined  $a_1 \equiv \psi - \beta(\psi + d)(1 - p_F)$ ,  $a_2 \equiv \psi - \beta(\psi + d_\kappa)(1 - p_F)$ ,  $a_3 \equiv \beta p_F$ ,  $a_4 \equiv \psi + d_\kappa$ , and  $a_5 \equiv \psi + d$ . Notice that  $a_i > 0$  for all  $i$ ,  $a_1 < a_2$ , and  $a_4 < a_5$ . The first order-conditions in (a.5) with respect to  $t_{HF}, t_{FF}$ , are

$$-a_1 + a_3 u'(a_4 t_{HF} + a_5 t_{FF}) a_4 \leq 0, \quad = 0 \text{ if } t_{HF} > 0, \quad (\text{a.6})$$

$$-a_2 + a_3 u'(a_4 t_{HF} + a_5 t_{FF}) a_5 \leq 0, \quad = 0 \text{ if } t_{FF} > 0. \quad (\text{a.7})$$

Hence, the optimal choice of  $t_{HF}, t_{FF}$  is a corner solution, with the exception of the knife-edge case in which  $a_2/a_1 = a_5/a_4$ . If  $a_2/a_1 > a_5/a_4$ , then  $t_{HF} > 0$  and  $t_{FF} = 0$ . On the other hand, if  $a_2/a_1 < a_5/a_4$ , then  $t_{HF} = 0$  and  $t_{FF} > 0$ .

Using the definitions of the  $a_i$  terms, one can show that  $a_2/a_1 < a_5/a_4$  if and only if

$$\beta(1 - p_F)(2d - \kappa) < \psi [1 - 2\beta(1 - p_F)]. \quad (\text{a.8})$$

First, notice that if  $\beta \geq 1/(2(1 - p_F))$  (region  $R_4$  in Figure 1), the condition in (a.8) can never hold, since the left-hand side is positive. In this case,  $t_{FF} = 0$  and the value of  $t_{HF}$  is given by (a.6), which after replacing for the  $a_i$  terms is equivalent to (9).

From now on let  $\beta < 1/(2(1 - p_F))$ . The condition in (a.8) becomes

$$\psi > \frac{\beta(1 - p_F)(2d - \kappa)}{1 - 2\beta(1 - p_F)} \equiv \psi_c. \quad (\text{a.9})$$

Using the definition of  $\psi_c$  above and recalling the definition of the “fundamental value” of the asset,  $\psi^* \equiv \beta d/(1 - \beta)$ , one can show that  $\psi^* > \psi_c$  if and only if

$$\kappa(1 - \beta)(1 - p_F) > d(1 - 2p_F). \quad (\text{a.10})$$

After some algebra, it turns out that the inequality in (a.10) holds if either: i)  $p_F \geq 1/2$  (region  $R_1$  in Figure 1) or ii)  $p_F < 1/2$ ,  $\beta < p_F/(1 - p_F)$  and  $\kappa/d > (1 - 2p_F)[(1 - \beta)(1 - p_F)]^{-1} \equiv \tilde{\kappa}$  (the values of  $\beta$  and  $p_F$  described here are represented by region  $R_2$  in Figure 1). However, Lemma 3 indicates that any equilibrium price satisfies  $\psi \geq \psi^*$ . Therefore, if  $\psi^* > \psi_c$ , the inequality in (a.9) is always true. In this case,  $t_{HF} = 0$  and the value of  $t_{FF}$  is given by (a.7). After replacing for the  $a_i$  terms, this is equivalent to (8).

We are left with the case in which either  $\beta, p_F$  fall in region  $R_3$  of Figure 1 or they fall in region  $R_2$  and also  $\kappa/d \leq \tilde{\kappa}$ . Here,  $\psi^* \leq \psi_c$ , and there will exist equilibrium prices that can be larger or smaller than  $\psi_c$  (this will be determined later when we introduce the supply of the assets,  $T$ ). For  $\psi$ 's that are large enough, so that (a.9) holds, the optimal choice of the agent satisfies  $t_{HF} = 0$ ,  $t_{FF} > 0$ . For  $\psi \in [\psi^*, \psi_c)$ , we have  $t_{HF} > 0$ ,  $t_{FF} = 0$ . In the knife-edge case where  $\psi = \psi_c$ , we have  $t_{HF}, t_{FF} > 0$ , and both (a.6), (a.7) hold with equality. The choices of  $t_{HF}, t_{FF}$  cannot be separately pinned down, but they are such that the quantity bought in the DM satisfies  $u'(q_F) = p_F/(1 - p_F)$ .

*Step 5:* In this step we establish the statements of Lemma 5 for  $\psi > \psi^*$  (which we have assumed so far). The case in which  $\psi = \psi^*$  is discussed in Step 6. Let us summarize the results derived so far. From Step 1, we know that the agent will never choose  $t_{HF}, t_{FF}$  in region I. From Step 2, we know that if the agent wishes to purchase  $q_F > \underline{q}(\psi)$ , she will do so by using  $t_{FF}$  only. From Step 3, we know that points in the interior of region III are dominated by asset holdings that purchase the same quantity, i.e.  $\underline{q}(\psi)$ , using fewer home assets. Finally, from Step 4, we know that if the agent wishes to purchase  $q_F \leq \underline{q}(\psi)$ , she will do so by using either  $t_{FF}$  or  $t_{HF}$ , but not both, as a medium of exchange.

Case 1: If parameter values are as in Case 1 of the lemma, the agent uses only  $t_{FF}$  as a medium of exchange. Hence, the objective function is given by (a.2), and the optimal choice of  $t_{FF}$  is described by (8).

Case 2: Next, consider parameter values as in Case 2 of the lemma. This case is less straightforward than Case 1. We have shown (Step 4) that if the agent wishes to purchase some  $q_F \leq \underline{q}(\psi)$ , she is better off by setting  $t_{FF} = 0$ . But from Step 1 we also know that if  $q_F > \underline{q}(\psi)$  the agent is better off using the foreign asset as a medium of exchange. To establish Case 2 of

the lemma, we need to exclude the latter possibility. Suppose, by way of contradiction, that the agent wants to purchase  $q_F > \underline{q}(\psi)$  and, therefore, chooses  $t_{FF} > 0$ . Under the contradictory assumption, the quantity of special good purchased is

$$q_F(\psi) \equiv \left\{ q : \psi = \beta [(1 - p_F)(\psi + d_\kappa) + p_F(\psi + d) u'(q)] \right\}. \quad (\text{a.11})$$

Notice three important facts. First,  $q_F(\psi)$  is strictly decreasing in  $\psi$ . Second,  $\underline{q}(\psi)$  is strictly increasing in  $\psi$ . Third, we claim that  $q_F(\psi^*) < \underline{q}(\psi^*)$ . To see why this is true, observe that

$$\begin{aligned} u'(\underline{q}(\psi^*)) &= \frac{d}{d - \kappa(1 - \beta)}, \\ u'(q_F(\psi^*)) &= 1 + \frac{\kappa(1 - p_F)(1 - \beta)}{dp_F}. \end{aligned}$$

Our claim that  $q_F(\psi^*) < \underline{q}(\psi^*)$  will be true if and only if  $u'(q_F(\psi^*)) > u'(\underline{q}(\psi^*))$ , which is true if and only if  $1 + \kappa(1 - p_F)(1 - \beta)/(dp_F) > d/[d - \kappa(1 - \beta)]$ , which after some more manipulations can be written as

$$\frac{\kappa}{d} < \frac{(1 - 2p_F)}{(1 - p_F)(1 - \beta)}. \quad (\text{a.12})$$

Since here  $\beta \geq p_F/(1 - p_F)$ , the right-hand side of (a.12) is greater than or equal to 1. Hence, the inequality in (a.12) is always satisfied. Combining these three facts implies that  $q_F(\psi) < \underline{q}(\psi)$  for all  $\psi$ , which is a straightforward contradiction to our assumption that the agent purchases  $q_F > \underline{q}(\psi)$  and, therefore, chooses  $t_{FF} > 0$ . Hence, the agent chooses to use only the home asset as a medium of exchange, and (given that  $\psi > \psi^*$ ) the optimal  $t_{HF}$  solves (9).

**Case 3:** Finally, consider parameters such as in Case 3 of the lemma. We know that if the agent wishes to purchase  $q_F \leq \underline{q}(\psi)$ , we have  $t_{FF} > 0$  if and only if  $\psi \geq \psi_c$ . Using an argument identical to the one we used in Case 2, one can show that the agent will never wish to purchase  $q_F > \underline{q}(\psi)$ .<sup>27</sup> Thus, the analysis in Step 4 fully characterizes the optimal choice of the agent, and Case 3 of the lemma follows directly.

*Step 6:* We only need to conclude the description of the optimal choice of  $t_{HF}$  when  $\psi = \psi^*$ . In Case 1, the home asset is not used as a medium of exchange. If  $\psi = \psi^*$ , the cost of carrying the home asset is zero. Hence, any  $t_{HF} \in \mathbb{R}_+$  is optimal. In Case 2, the buyer uses the home asset as a means of payment. Thus, with  $\psi = \psi^*$ , she is willing to carry any amount of  $t_{HF}$  that allows her to buy the  $q_F$  which maximizes the buyer's surplus in the foreign DM. This quantity is given by  $\underline{q}(\psi^*)$ , implying that any  $t_{HF} \geq \underline{q}(\psi^*)/(d/(1 - \beta) - \kappa)$  is optimal. Finally, in Case 3,  $\psi = \psi^*$  means that  $\psi < \psi_c$ . Therefore, the optimal behavior of the buyer coincides with Case 2 described above.  $\square$

<sup>27</sup> This follows directly from (a.12), since the term on the right-hand side is just  $\tilde{\kappa}$ .

*Proof of Proposition 1.* The fact that  $t_{FH} = 0$  follows immediately from Lemma 4. The proofs for parts (a), (b), and (c) are similar. Hence, we only prove part (a) in detail.

The fact that agents use only foreign assets in order to trade in the foreign  $DM$  follows from Lemma 5. Moreover, applying total differentiation in (7) and (8) yields

$$\begin{aligned}\frac{\partial q_H}{\partial \psi} &= \frac{d}{\beta p_H (\psi + d)^2 u''(q_H)} < 0, \\ \frac{\partial q_F}{\partial \psi} &= \frac{d - \kappa \beta (1 - p_F)}{\beta p_F (\psi + d)^2 u''(q_F)} < 0.\end{aligned}$$

Since  $q_H$  and  $q_F$  are strictly decreasing in  $\psi$ , there exists a critical level of  $T$ ,  $T_1^*$ , such that  $T \geq T_1^*$  implies  $\psi = \psi^*$ , which in turn implies  $q_H = q^*$  and  $q_F = q_{F,1}^*$  (the latter statement follows from (7) and (8)). To find this critical level use a) the bargaining solutions  $(\psi + d)t_{HH} = q_H$ ,  $(\psi + d)t_{FF} = q_F$ , evaluated at  $\psi = \psi^*$ , and b) the market clearing condition, which under the specific parameter values becomes  $T = t_{HH} + t_{FF}$ . This yields  $T_1^* = (1 - \beta)(q^* + q_{F,1}^*)/d$ . If  $T > T_1^*$ , the demand of foreigners for a certain asset is given by  $t_{FF} = (1 - \beta)q_{F,1}^*/d$ . This allows them to purchase the maximum possible quantity of foreign special good,  $q_{F,1}^*$ . The rest of the supply,  $T$ , is absorbed by local buyers. Hence,  $t_H = t_{HH} = T - (1 - \beta)q_{F,1}^*/d$ .

When  $T < T_1^*$ , agents are not buying the maximum possible amount of the special good. Hence, in this range,  $\psi > \psi^*$ ,  $q_H < q^*$ ,  $q_F < q_{F,1}^*$ , and  $\partial \psi / \partial T < 0$ ,  $\partial q_H / \partial T > 0$ ,  $\partial q_F / \partial T > 0$ . As  $T \rightarrow T_1^*$ , the liquidity properties of the asset are exploited;  $\psi$  reaches the fundamental value,  $\psi \rightarrow \psi^*$ , and  $q_H, q_F$  reach their upper bounds, namely  $q_H \rightarrow q^*$  and  $q_F \rightarrow q_{F,1}^*$ .  $\square$

*Proof of Lemma 6.* a) Let  $T < T_1^*$ . i) The monotonicity of  $q_H(p_H)$  and  $q_F(p_F)$  follows from the fact that  $u'' < 0$ . The monotonicity of  $t_H(p_H)$  and  $t_F(p_F)$  follows trivially from the bargaining solution. ii) For some  $p_H = p_F = p \in (0, 1)$ ,  $p_H(p) > p_F(p)$  will be true if and only if  $u'(q_H) < u'(q_F)$ . After some algebra, one can show that this is equivalent to  $0 < \kappa(1 - p)$ , which is always true. iii) The fact that  $\tilde{p}_H(p_F) < p_F$  follows directly from parts (i) and (ii). Given the definition of  $\tilde{p}_H(p_F)$  and the fact that  $q_H(p_H)$  is increasing, for any  $p_F$  and  $p_H > \tilde{p}_H(p_F)$ , we have  $q_H(\tilde{p}_H(p_F)) > q_F(p_F)$ . iv) The result follows directly from part (iii) and the bargaining solution.

b) Let  $T \geq T_1^*$ . The facts  $q_H = q^*$ ,  $q_F = q_{F,1}^*$ ,  $t_H = T - q_{F,1}^*(1 - \beta)/d$ , and  $t_F = q_{F,1}^*(1 - \beta)/d$  are already presented in Proposition 1. Also,  $q^* > q_{F,1}^*$  follows directly from the definition of  $q_{F,1}^*$ . To see why  $T - q_{F,1}^*(1 - \beta)/d > q_{F,1}^*(1 - \beta)/d$ , recall that here

$$T > T_1^* = \frac{1 - \beta}{d} (q^* + q_{F,1}^*) > \frac{2(1 - \beta)}{d} q_{F,1}^*,$$

where the last inequality uses the fact that  $q^* > q_{F,1}^*$ . This concludes the proof.  $\square$

*Proof of Proposition 2.* We prove that the expressions on the left-hand sides of (10) and (11) are greater than 0.5. The derivation of these expressions is relegated to Appendix B.

- a) If for a given  $p_F, p_H > \tilde{p}_H(p_F)$ , the inequality in (10) follows immediately from Lemma 6.  
b) The left-hand side of (11) is bigger than 0.5 if and only if

$$T > \frac{1 - \beta}{d} q_{F,1}^* (2 - p_F).$$

This is always satisfied since, by Lemma 6,  $t_H = T - q_{F,1}^* (1 - \beta)/d > t_F = q_{F,1}^* (1 - \beta)/d$ .  $\square$

## B Accounting Appendix

Although agents are ex-ante identical, their decisions in the *CM*, regarding how many hours to work or whether they should visit the financial markets and re-balance their portfolios, depend on whether they got matched in the local and/or foreign *DM*. In this Appendix, we focus on the labor, consumption, and asset-holding decisions of the following seven groups: 1) sellers who got matched with a local buyer (group 1), 2) sellers who got matched with a foreign buyer (group 2), 3) sellers who did not get matched (group 3), 4) buyers who got matched in both *DM*'s (group 4), 5) buyers who got matched only in the home *DM* (group 5), 6) buyers who got matched only in the foreign *DM* (group 6), and 7) buyers who did not get matched (group 7).

The measures of these groups are given by:  $\mu_1 = \xi a_S \sigma_H / (\sigma_H + \sigma_F)$ ,  $\mu_2 = \xi a_S \sigma_F / (\sigma_H + \sigma_F)$ ,  $\mu_3 = \xi(1 - a_S)$ ,  $\mu_4 = \sigma_H \sigma_F a_B^2$ ,  $\mu_5 = \sigma_H a_B (1 - \sigma_F a_B)$ ,  $\mu_6 = \sigma_F a_B (1 - \sigma_H a_B)$ , and  $\mu_7 = (1 - \sigma_H a_B)(1 - \sigma_F a_B)$ . For future reference notice that  $\mu_1 = p_H$ ,  $\mu_2 = p_F$ ,  $\mu_4 = p_H p_F$ ,  $\mu_5 = p_H(1 - p_F)$ ,  $\mu_6 = p_F(1 - p_H)$ , and  $\mu_7 = (1 - p_H)(1 - p_F)$ .

*Derivation of Home Assets Bias formulas in (10) and (11).* We define asset home bias as the ratio of the weighted sum of domestic asset holdings over the weighted sum of all asset holdings for all citizens of a certain country. We count asset holdings in both the *CM* and the *DM* and assume equal weights for the two markets. It is understood that we count asset holdings at the end of the sub-periods. For example, an agent of group 4 enters the *CM* with no assets, but re-balances her portfolio, so at the end of the *CM* she holds  $t_H$  of the home asset and  $t_F$  of the foreign asset. On the other hand, an agent of group 1 enters the *CM* with some home assets, but she sells these assets, so at the end of the *CM* she holds zero.

a) First consider  $T \leq T_1^*$ . Let the numerator of the asset home bias formula (weighted sum of domestic asset holdings) be denoted by  $A_H$ . Given the strategy described above,

$$A_H = t_H + p_H t_F + p_F t_F + (1 - p_H) t_H.$$

The first term is the total home asset holdings in the *CM*, i.e.  $t_H$  times the total measure of buyers (one). Sellers hold zero assets in the *CM*. The second term represents home asset

holdings in the  $DM$  by sellers who got matched with local buyers. The third term represents home asset holdings in the  $DM$  by sellers who got matched with foreign buyers. Recall that we are describing the local asset dominance regime. Hence, foreign buyers pay sellers in local assets. The last term represents home asset holdings in the  $DM$  by the buyers who did not get matched in the local  $DM$  and, hence, hold some domestic assets (groups 6 and 7). After some algebra one can conclude that  $A_H = 2t_H + p_F t_F$ .

Following similar steps, one can show that the denominator of the asset home bias formula (weighted sum of all asset holdings), is given by  $A_T = 2(t_H + t_F)$ . From market clearing,  $t_H + t_F = T$ . Therefore, (10) follows immediately.

b) Consider  $T > T_1^*$ . Now the cost of carrying home assets is zero. As a result, buyers carry more assets than what they use in the  $DM$  as media of exchange. Therefore, agents from groups 4 and 5, who previously did not show up in the expression  $A_H$ , will now carry some home asset leftovers. The numerator of the asset home bias formula is

$$\begin{aligned} A_H &= \left( T - \frac{1-\beta}{d} q_{F,1}^* \right) + p_H \frac{1-\beta}{d} q^* + p_F \frac{1-\beta}{d} q_{F,1}^* \\ &+ p_H \left( T - \frac{1-\beta}{d} q^* - \frac{1-\beta}{d} q_{F,1}^* \right) + (1-p_H) \left( T - \frac{1-\beta}{d} q_{F,1}^* \right). \end{aligned}$$

The first term is the total home asset holdings in the  $CM$  (held by buyers whose measure is one). The second term represents home asset holdings in the  $DM$  by sellers who got matched with local buyers. The third term represents home asset holdings in the  $DM$  by sellers who got matched with foreign buyers. The fourth term represents home asset holdings in the  $DM$  by buyers who got matched in the local  $DM$  (groups 4 and 5). Finally, the fifth term stands for the home asset holdings of buyers who did not trade in the local  $DM$  (groups 6 and 7). After some algebra, we obtain

$$A_H = 2 \left( T - \frac{1-\beta}{d} q_{F,1}^* \right) + p_F \frac{1-\beta}{d} q_{F,1}^*.$$

One can also show that the denominator of the asset home bias formula is given by  $A_T = 2T$ . Hence, (11) follows immediately.  $\square$

*Proof of Proposition 3.* a) Let  $C_F^i, C_H^i$  be the value of foreign and domestic consumption for the typical agent in group  $i, i = 1, \dots, 7$ , respectively. All consumption is denominated in terms of the general good. Moreover, let  $H^i$  denote the hours worked by the typical agent in group  $i, i = 1, \dots, 7$ . All agents consume  $X^*$  in the  $CM$ , so agents with many assets will work fewer hours. Sometimes it will be useful to directly use the agents' budget constraint and replace  $X^*$  with more informative expressions. First, consider sellers. We have  $C_H^1 = C_H^2 = C_H^3 = X^*$  and  $C_F^1 = C_F^2 = C_F^3 = 0$ . Clearly, even sellers who matched with foreign buyers have no imported

consumption, since in the local asset dominance regime, sellers get paid in local assets.

Unlike sellers, buyers might also consume in the  $DM$ . For group 4, we have  $C_H^4 = X^* + (\psi + d)t_H = X^* + q_H$ , where  $q_F$  represents  $DM$  consumption. Also,  $C_F^4 = (\psi + d)t_F = q_F$ . Group 5 does not match abroad, hence these agents carry some foreign assets into the  $CM$ , which in turn implies that some of their  $CM$  consumption is imported. Clearly,  $C_F^5 = d_\kappa t_F$ . On the other hand,  $C_H^5 = H^5 + dt_H$ .<sup>28</sup> For group 6, all  $CM$  consumption ( $X^*$ ) is domestic and all  $DM$  consumption,  $q_F$ , is imported. Finally, group 7 consumes only in the  $CM$ . We have  $C_H^7 = H^7 - dt_H$  and  $C_F^7 = d_\kappa t_F$ .

The analysis in the previous paragraph reveals that  $p_H > \tilde{p}_H(p_F)$ , implies  $C_H^i > C_F^i$  for all groups except 6, for which the inequality could be reversed, depending on the magnitude of  $X^*$ . To prove the desired result we contrast the domestic and foreign consumption for groups 2 and 6.  $C_H > C_F$  will be true if and only if  $\sum_{i=1}^7 \mu_i C_H^i > \sum_{i=1}^7 \mu_i C_F^i$ . Since  $C_H^i > C_F^i$  holds for  $i = 1, 3, 4, 5, 7$ , the latter statement will be true as long as

$$\mu_2 C_H^2 + \mu_6 C_H^6 \geq \mu_2 C_F^2 + \mu_6 C_F^6. \quad (\text{b.1})$$

From the budget constraint of group 2, we have  $C_H^2 = X^* = H^2 + q_F$ . Also, from the analysis above,  $C_F^2 = 0$ . Hence, (b.1) holds if and only if

$$\begin{aligned} p_F(H^2 + q_F) + p_F(1 - p_H)X^* &\geq p_F(1 - p_H)q_F \Leftrightarrow \\ p_F H^2 + p_F(1 - p_H)X^* &\geq -p_H p_F q_F, \end{aligned}$$

which is always true, since  $X^*, H^i \geq 0$ .

b) Like in part (a),  $C_H^1 = C_H^2 = C_H^3 = X^*$  and  $C_F^1 = C_F^2 = C_F^3 = 0$ . For group 4  $C_H^4 = X^* + q^*$  and  $C_F^4 = q_{F,1}^*$ . For group 5  $CM$  consumption ( $X^*$ ) can be broken down into domestic,  $H^5 + dT - q^* - (1 - \beta)q_{F,1}^*$ , and imported,  $d_\kappa(1 - \beta)q_{F,1}^*/d$ . In the  $DM$  all consumption is domestic and equal to  $q^*$ . Hence,  $C_H^5 = H^5 + dT - (1 - \beta)q_{F,1}^*$  and  $C_F^5 = d_\kappa(1 - \beta)q_{F,1}^*/d$ . For group 6 all  $CM$  consumption ( $X^*$ ) is domestic and all  $DM$  consumption,  $q_{F,1}^*$ , is imported. Finally, group 7 only consumes in the  $CM$ . From this group's budget constraint, it follows that  $C_H^7 = H^7 + dT - (1 - \beta)q_{F,1}^*$  and  $C_F^7 = d_\kappa(1 - \beta)q_{F,1}^*/d$ .

It is clear that for  $i = 1, 2, 3, 4$ ,  $C_H^i > C_F^i$ . For groups 5 and 7 the same argument is true, but it is less obvious, hence we prove it formally below. For group 6 this argument need not be true. Consider group 5.  $C_H^5 > C_F^5$  will be true if and only if

$$H^5 + dT > (1 - \beta)q_{F,1}^* \left(2 - \frac{\kappa}{d}\right). \quad (\text{b.2})$$

<sup>28</sup> To see this point, notice that this agent's budget constraint is  $X^* + \psi(t_H + t_F) = H^5 + (\psi + d_\kappa)t_F$  or  $X^* = H^5 - \psi t_H + d_\kappa t_F$ , where  $d_\kappa t_F$  is imported  $CM$  consumption and  $H^5 - \psi t_H$  is domestic  $CM$  consumption. Since domestic  $DM$  consumption is given by  $(\psi + d)t_H$ , we have  $C_H^5 = H^5 + dt_H$ .

But here  $T > T_1^*$ , which implies

$$dT > (1 - \beta)(q^* + q_{F,1}^*) \Leftrightarrow dT > 2(1 - \beta)q_{F,1}^* \Leftrightarrow dT + H^5 > \left(2 - \frac{\kappa}{d}\right)(1 - \beta)q_{F,1}^*.$$

This confirms that the inequality in (b.2) holds true. Following similar steps establishes that  $C_H^7 > C_F^7$ . Hence,  $C_H^i > C_F^i$  for all  $i$ , except possibly group 6. However, as in part (a), one can easily verify that  $\mu_2 C_H^2 + \mu_6 C_H^6 \geq \mu_2 C_F^2 + \mu_6 C_F^6$ , which implies that  $C_H > C_F$  also holds.  $\square$