# Economic Stabilizers in Emerging Markets: The Case for Trade Credit*

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**Abstract**

We study the interplay between bank and trade credit in emerging markets. We document that small and medium-sized enterprises (SMEs) trade off bank for trade credit, while large firms are more likely to extend trade credit, especially during financial crises. We develop a model of heterogeneous firms that extend state-contingent credit to each other along supply chains for the purpose of providing insurance in the case of adverse economic shocks. The model predicts that firms obtain more trade credit the less bank credit they have available, the larger is their scale of operation, and the more-debt constrained they are relative to their trading partner. Further, more debt-constrained firms receive more state-contingent trade credit from their less-constrained partners. We validate the model's predictions using firm-level data from Hungary. We conclude that the insurance channel of trade credit earns it a role of a macroeconomic stabilizer in emerging markets.

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# 1 Introduction

Firms in emerging markets rely extensively on non-traditional sources of financing in the absence of a well-developed financial sector. In particular, it is very common for firms to finance their day-to-day operations via trade credit, which refers to inter-firm lending along supply chain lines (see Allen et al. (2013) for cross-country evidence on sources of financing). In this paper, we study the interplay between bank and trade credit. Using data for Mexico, we document several interesting facts. First, for small and medium-sized enterprises (SMEs), bank and trade credit act as substitutes, as reliance on trade credit increases when bank credit declines. Second, large firms are more likely to extend trade credit, including at longer maturities, when SMEs receive less bank credit. Third, during the great financial crisis (GFC) of 2008, listed firms in Mexico increase the share of their financial resources which are used to support trade credit lending. To reconcile these facts, we develop a model of heterogeneous firms that extend state-contingent credit to each other along supply chains for the purpose of providing insurance in the case of adverse economic shocks. The model predicts that firms obtain more trade credit the less bank credit they have available, the larger are their operations, and the more debt-constrained they are, relative to their trading partner. Furthermore, the volatility of trade credit receipts is higher for more debt-constrained firms, especially in comparison to their partners, which confirms the role that trade credit plays in diversifying away idiosyncratic risks. We verify that these predictions are supported by firm-level data from Hungary. Hence, we argue that trade credit acts as a macroeconomic stabilizer in emerging markets.

The theory that we develop features intermediate good producers and final good producers, both of whom are financially constrained and differentiated by their cost of borrowing from a bank. Intermediate good producers have to finance their cost of labor prior to production, while final good producers have to expense their purchases of intermediate goods prior to producing and delivering final goods to consumers. Final good producers face demand uncertainty, which affects intermediate good producers via the supply chain.

Producers potentially have access to two types of markets. With some positive probability, they may meet in a decentralized market where they exchange intermediate goods. In this setting, producers may opt to issue trade credit to each other in order to supplement bank loans needed to pay the upfront costs. Intermediate (final) good producers who refuse the terms of a bilateral meeting as well as those who do not get the opportunity to match can sell (buy) the intermediate good to (from) an intermediary at a perfectly competitive price. However, the intermediary requires that all purchases are paid for in advance and does not offer trade credit. Hence, producers have to rely entirely on bank borrowing in order to produce in this case.

The amount of trade credit that firms extend to each other depends critically on the firms' ability to borrow from a bank. The more asymmetric are the costs of borrowing for two parties, the more trade credit will be extended. More debt-constrained producers obtain more trade credit from less debt-constrained ones in order to achieve an efficient scale of production. Similarly, suppliers are willing to extend more trade credit to final good producers in a bad state of the world in order to prevent default.

We derive several testable predictions of the model. First, the model predicts that firms with less bank debt and as well as firms with larger scales of operation obtain more trade credit. The reasoning is simple: bank and trade credit are substitutes, so more debt-constrained producers need more trade credit. Conditional on the total amount of debt, firms that run larger operations need more trade credit in order to achieve an efficient scale. Furthermore, firms obtain more trade credit the lower is their total debt relative to their trade partner's, which reflects the relative borrowing constraints of the two agents. Additionally, the volatility of trade credit received is higher for smaller and more debt-constrained producers because they are in more need of state-contingent trade credit. Finally, the volatility of trade credit increases in the asymmetry of debt levels between two parties. Since trade credit is state contingent, this finding implies that less constrained agents offer more insurance to more constrained agents via trade credit. We verify these predictions using firm-level data from ORBIS for Hungary during the 2009-2014 period.

Our theoretical and empirical results suggest that trade credit mitigates idiosyncratic risks. The argument that less-constrained (often larger) firms extend trade credit to their partners in order to insure them in bad states of the world is supported by existing empirical studies. Hardy and Saffie (2019) find that stock-market listed firms in Mexico exploit cheap dollar funding to provide trade credit dominated in peso to related parties. In fact, we show that a generalization of our trade credit theory that features debt denominated in domestic and in foreign currency rationalizes this observation. In addition, Hardy and Saffie (2019) find that, during the peso depreciation that followed the Lehman Brothers' collapse, listed firms which experienced a balance sheet shock contracted their investment more than other firms, but did not decrease their trade credit lending more than other firms. This finding suggests that large firms protected their value chains by absorbing most of the exchange rate shock. The insurance channel that underlines the basis of our theory of trade credit offers a justification for this empirical result. In sum, our theory of state-contingent trade credit implies that large firms do not fully pass through adverse macroeconomic shocks onto SMEs. Hence, not only does trade credit act as a macroeconomic stabilizer in emerging markets, but also large-firm borrowing in USD may be more beneficial to a typical developing country than previously thought.

The remainder of the paper proceeds as follows: After describing the related literature, in Section 2, we present basic facts on the trade off between bank and trade credit. In Section 3, we outline our novel theory of trade credit. In Section 4, we explore the theoretical predictions of the model. In Section 5, we present the empirical results. We conclude in Section 6. In Appendix A, we outline and solve the general model with domestic- and foreign-currency debt, and we relegate all derivations and proofs to Appendix B.

**Related Literature.** Our work fits in the literature that studies the substitution between bank and trade credit within the context of financially-constrained firms. Love et al. (2007) find that tighter monetary policy is associated with more trade credit (both accounts payable and accounts receivable). They make the implicit connection that trade credit is replacing bank credit. Within the context of emerging markets, Choi and Kim (2005) document that trade credit increases in the immediate aftermath of financial crises, but falls in subsequent years. The decline is driven by more financially vulnerable firms. The authors argue that bank credit is (generally) redistributed into the economy from financially strong firms who can borrow from banks to financially weak firms who cannot in the form of trade credit. Minetti et al. (2018) find that firms exposed to bank credit rationing and with weak bank relationships participate more intensely in supply chains, particularly global supply chains (rather than internalizing production steps). Shapiro et al. (2018) also find a trade off between bank and trade credit, and study how trade credit affects firm pricing dynamics. Petersen and Rajan (1997) document several findings including the fact that small firms rely on trade credit when bank credit is unavailable, firms with access to (bank) credit offer more trade credit, and suppliers extend more trade credit to financially constrained firms. They argue that supplier firms have an informational advantage over banks regarding the state of their trade partner and could recover a larger fraction of the loan in the event of a default. Like these empirical papers, we also find that trade and debt credit are substitutes. In addition, we make a theoretical contribution to this literature and we introduce a novel reason for the existence of trade credit, which is the provision of insurance between trading partners.

Alfaro et al. (2021) find that credit supply shocks can propagate downstream through production networks, both via price and the trade credit extended by suppliers. Kalemli-Ozcan et al. (2014) develop a model of production chains that predicts that more trade credit is supplied by firms that are more upstream, but these upstream firms are more sensitive to changes in the availability of credit. The model implies that shocks to bank credit can amplify its real impacts via production chains. Unlike these papers, we find that trade credit mitigates shocks through production networks and therefore trade credit acts as a macroeconomic stabilizer.

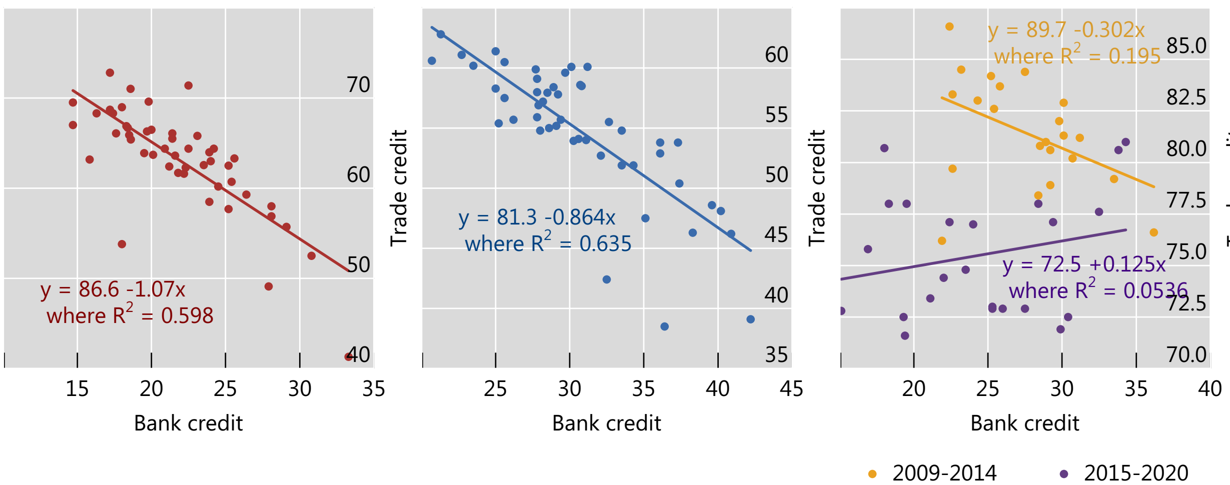
There is a growing literature that offers theoretical justifications for the existence of trade credit. In a seminal paper, Burkart and Ellingsen (2004) develop a model where trade credit co-exists with bank debt due to the ability of suppliers to better monitor trading partners who try to renege on their contract. Giannetti et al. (2011) follow up by documenting empirical regularities about the characteristics of the types of firms that participate in trade credit. Klapper et al. (2011) argue that non-financial factors can drive trade credit terms, such as larger firms having bargaining power over smaller firms. Giannetti et al. (2021) argue that trade credit is used to maintain customer relationships. The paper emphasizes the bargaining power of large firms and their small suppliers who use trade credit to give trade surplus to the large firms without engaging in price discrimination or cannibalization of their sales to other firms. Finally, Garcia-Marin et al. (2020) study trade credit within a framework of reputation building. We differ from this literature in that we examine a new role for trade credit; that of providing insurance to trade partners, thus stabilizing the economy.

# 2 Trade Credit Facts

In this section, we use data from Mexico to establish basic facts about the role that trade credit plays in a typical emerging market. We begin by studying the trade off between bank credit and trade credit for SMEs, which don't have access to a broad range of financial instruments. In order to examine this relationship for SMEs, we turn to the Credit Market

Survey conducted quarterly by the Banco de Mexico. This survey asks at least 450 firms across Mexico about their access to different forms of credit, accessibility of bank credit, as well as their extension of trade credit to other firms. It then provides aggregated responses by firm size. There is a structural break in the survey around 2009, the time of the great financial crisis (GFC), so we examine the pre- and post-GFC data separately.

Figure 1: Trade-off of bank and trade credit for SMEs




(A) Small firms, 1998Q1-2008Q2

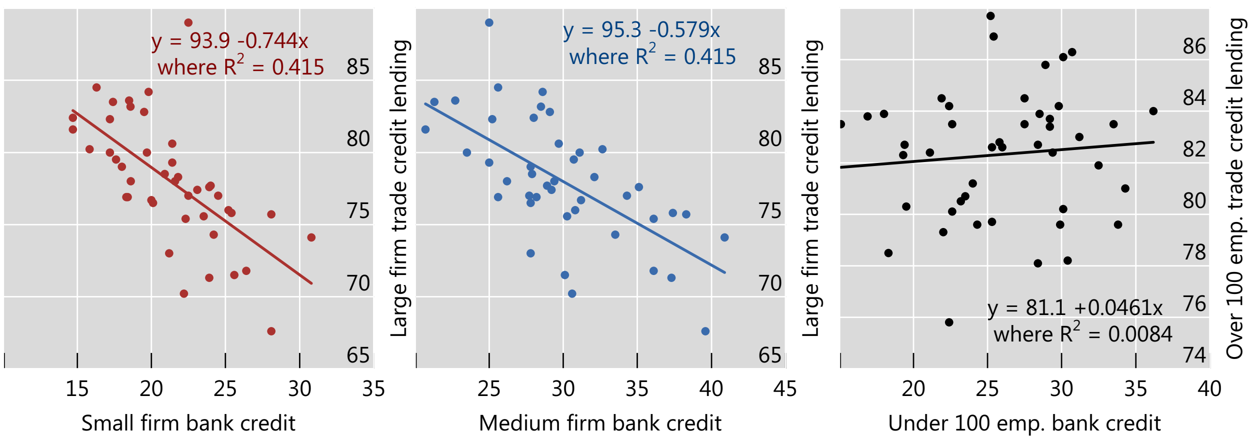
(B) Medium firms, 1998Q1-2008Q2

(C) Firms with <100 employees, 2009Q3-2020Q3

Percent of firms receiving bank credit vs. percent of firms receiving trade credit (or for pre-2009, whose main credit source is trade credit). Small firms have 1997 sales between 1-100 million pesos. Medium firms have 1997 sales between 101-500 million pesos. Source: Credit market survey, Banco de Mexico.

**Fact 1: SMEs trade off bank credit for trade credit.** Figure 1 illustrates the trade-off between bank and trade credit for SMEs. The left and center panels show that, as the proportion of SMEs with any bank credit declines, the proportion listing trade credit as their most important source of credit rises. In the post-GFC survey in the right panel, we see that, for the first 5 years after the GFC, as the share of firms with any bank credit declines, the share of firms with any trade credit (not just those for whom it is most important) increases. The latter half of the sample (purple dots) shows that this relationship is not always a strict trade-off. Indeed, during periods of growth we may expect firms to increase their access to both bank credit and trade credit (although the purple slope is not statistically significant). Further, these surveys may miss the relationship between the volume of trade or bank credit that firms receive. Nevertheless, there appears to be a relevant trade off in use or importance of trade credit for SMEs.

Figure 2: Trade credit lending by large firms and bank credit of SMEs




(a) Small firms, 1998q1-2008q2

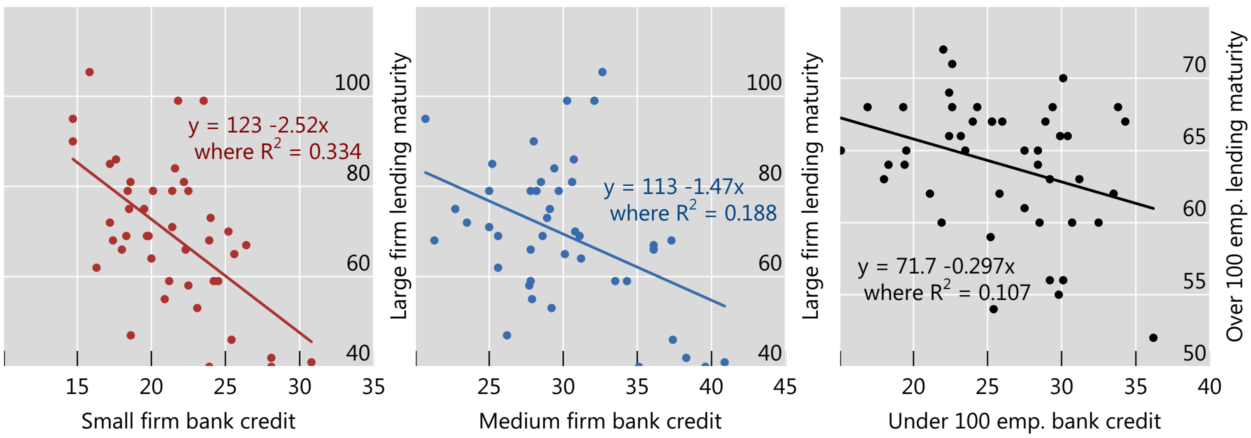
(b) Medium firms, 1998q1-2008q2

(c) Firms with <100 employees, 2009q3-2020q3

Percent of firms receiving bank credit vs. percent of large firms extending trade credit. Small firms have 1997 sales between 1-100 million pesos. Medium firms have 1997 sales between 101-500 million pesos. Large firms have 1997 sales between 501-5000 million pesos. Firms with more than 5000 million pesos in sales make up less than 4% of the survey sample, so their responses are excluded. Source: Credit market survey, Banco de Mexico.

**Fact 2: Large firms extend more trade credit when SMEs' bank loans decline.** Figure 2 examines how trade credit lending by large firms evolves as SMEs report less use of bank credit. Figures 2a and 2b show that, as the share of SMEs with any bank credit declines, the share of large firms extending trade credit to other firms increases. The survey does not reveal whether the share of trade credit lending to SMEs increases, but the trade credit networks of large firms expand at the same time as SMEs are receiving less credit from banks. Post-GFC, this relationship is less clear in terms of the share of large firms extending trade credit. However, the volume of trade credit extended by such firms could increase. Consistent with this, we observe in Figure 3 that the maturity of trade credit lending by large firms lengthens as SMEs' use of bank credit declines. This holds both pre- and post-GFC. Effectively, giving longer repayment terms for trade credit could increase the volume of trade credit between firms in a given quarter, in addition to easing the financing burden on the recipient firms.

Figure 3: Average maturity of trade credit granted by large firms vs bank credit of SMEs




(a) Small firms, 1998q1-2008q2

(b) Medium firms, 1998q1-2008q2

(c) Firms with <100 employees, 2009q3-2020q3

Percent of SMEs receiving bank credit vs. average maturity in days of trade credit lent by large firms. Small firms have 1997 sales between 1-100 million pesos. Medium firms have 1997 sales between 101-500 million pesos. Large firms have 1997 sales between 501-5000 million pesos. Firms with more than 5000 million pesos in sales make up less than 4% of the survey sample, so their responses are excluded. Source: Credit market survey, Banco de Mexico.

**Fact 3: Trade credit rises during adverse economic conditions.** To examine the trade credit lending of large firms in greater detail, we turn to detailed firm-level data for stock-market listed non-financial firms in Mexico. This dataset is derived from quarterly financial statements made by companies listed on the Mexican Stock Exchange (BMV) comprising 183 firms (unbalanced) over 2005q1-2015q2.[1] From this data, we have a detailed look

[1]See Hardy (2018) for more details of this dataset.

at the sources of financing for these firms (i.e. the structure of their liabilities) as well as how these resources are used (i.e. the structure of their assets).

Large firms have much broader access to credit than SMEs, and are able to tap into sources of credit even when credit becomes tighter for the general economy. This easier access to credit enables large firms to serve as a type of financial intermediary for other firms, borrowing from traditional sources of credit and then increasing their extension of trade credit to other firms (Hardy and Saffie, 2019).

We next examine the propensity of firms to use each source of their borrowing to finance trade credit, and if that relationship changes during the GFC when credit conditions tighten. We run the following regression:

$$\frac{\Delta Accounts\ Receivable_{it}}{Assets_{it-1}} = \alpha_i + \alpha_t + \sum_{j \in FS} \frac{\Delta Funding\ Source^j_{it}}{Assets_{it-1}}(\beta^j_1 + \beta^j_2 Crisis_t) + \epsilon_{it}$$

where $FS$ is the set of funding sources of the firm: net profits ("cash flow"), bonds, loans, trade credit, or other liabilities. $Crisis$ takes a value of 1 over 2008q3-2010q2.

The results are shown in Table 1. The interpretation of coefficients in this table is as follows: for every dollar increase that the firm receives via each source (e.g. \$1 more in loans), the coefficient indicates how much of this dollar is allocated towards trade credit lending (accounts receivables). For example, column (1) indicates that on average \$0.18 of a \$1 increase in bank loans finances the firms' extension of trade credit to other firms. This proportion during normal times (excluding the GFC) ranges from \$0.14 for internal cash flow from profits up to \$0.21 for credit received from other firms.

During the GFC, the propensity of firms to use these sources of credit to finance their own extension of trade credit increases. Specifically, an additional \$0.23 out of every dollar of profits goes to fund trade credit lending, as well as an additional \$0.31 from every dollar of new bond debt and \$0.11 of every new loan. Columns (2) and (3) break this relationship

Table 1: Corporate funding and the supply of trade credit

| | (1) Total | (2) Customers | (3) Others |
|---|---|---|---|
| Cash Flow | 0.141*** | 0.0930*** | 0.0482* |
| | (0.0386) | (0.0220) | (0.0283) |
| Δ Bond | 0.168*** | 0.0793** | 0.0890** |
| | (0.0540) | (0.0366) | (0.0381) |
| Δ Loan | 0.183*** | 0.127*** | 0.0554*** |
| | (0.0301) | (0.0261) | (0.0167) |
| Δ Trade Credit | 0.212*** | 0.187*** | 0.0249 |
| | (0.0415) | (0.0346) | (0.0304) |
| Δ Other Liab | 0.159*** | 0.0952*** | 0.0638** |
| | (0.0397) | (0.0226) | (0.0310) |
| Cash Flow × Crisis | 0.229*** | 0.199*** | 0.0303 |
| | (0.0654) | (0.0522) | (0.0455) |
| Δ Bond × Crisis | 0.312*** | -0.0800** | 0.392*** |
| | (0.0589) | (0.0391) | (0.0393) |
| Δ Loan × Crisis | 0.106* | 0.0196 | 0.0860*** |
| | (0.0630) | (0.0450) | (0.0323) |
| Δ Trade Credit × Crisis | 0.00193 | -0.0338 | 0.0355 |
| | (0.0731) | (0.0604) | (0.0453) |
| Δ Other Liab × Crisis | 0.0600 | 0.0137 | 0.0466 |
| | (0.0678) | (0.0513) | (0.0387) |
| Observations | 4771 | 4779 | 4771 |
| $R^2$ | 0.177 | 0.0633 | 0.222 |
| Firms | 183 | 183 | 183 |
| FirmFE | Yes | Yes | Yes |
| TimeFE | Yes | Yes | Yes |

Sample spans 2005q2-2015q2. Row labeled Firms reports the number of firms in each regression. Dependent variable is change in accounts receivable (either total, those to customers, or those to non-customers). Cash flow is net income over the previous quarter; Δ Bond is the change in bond debt over the previous quarter; Δ Loan is change in bank debt over the previous quarter; Δ Trade Credit is the change in trade credit liabilities (accounts payable) over the previous quarter. Δ Other is the change in all other liabilities (besides bank, trade, and bond credit) over the previous quarter. All variables are normalized by lagged assets. Crisis is a dummy taking a value of 1 over 2008q3-2010q2. Errors are clustered at the firm level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

down by trade credit to the firm's customers vs. trade credit to other non-customer firms (suppliers, related firms, other firms). We see that these large firms utilize their profits in order to provide financing to their customers, whereas any additional bond or loan borrowing is actually used to support trade credit to non-customers. These findings suggest that firms extend trade credit when debt supply dries up, which makes trade credit assume the role of a macroeconomic stabilizer. In the next section, we develop a theory where trade credit assumes this very role.

# 3 Theory of Trade Credit

The economy consists of three types of agents: a unit measure of producers of an intermediate good, a unit measure of producers of a final good, who use the intermediate good to produce and deliver a final product to the consumers, and a representative intermediary whose role is to ensure that all markets for the intermediate good clear. We label the intermediate good producer (who sells goods to the final good producer) as "seller" and we label the final good producer as "buyer". The time horizon consists of two periods. In period 1, there is uncertainty regarding the realization of price (alternatively, demand) for the final good, which is sold in period 2.

In period 1, sellers use labor in order to produce the intermediate good according to a production function $X = \ln L$, where $X$ denotes the quantity of intermediate good produced and $L$ denotes the amount of labor units employed at wage rate $w$. Sellers begin the period with zero net worth, so in order to hire labor, they need to raise funds. Sellers are heterogeneous in their borrowing ability. Let $s$ denote a given seller. They can borrow an amount $D_s$ from a bank, which needs to be repaid in period 2 at interest rate $r^*$. Any amount saved between period 1 and 2 earns the same rate of interest, $r^*$. The seller also incurs a borrowing cost $\psi_s D_s^2$, where higher values of $\psi_s > 0$ are associated with relatively more debt-constrained sellers. Alternatively, a seller may obtain credit from a final good producer to whom they sell the intermediate product, i.e. trade credit. Net trade credit can be negative; that is, a seller may opt to extend trade credit to a buyer. Buyers obtain intermediate goods from sellers in period 1 and transform them into final goods using a linear technology, where a unit of input yields a unit of final good. Like sellers, buyers begin period 1 with zero net worth and need to raise funds. Let $b$ denote a given buyer characterized by their borrowing cost $\psi_b > 0$. They can raise debt $D_b$ to be repaid in period 2 at interest rate $r^*$ while incurring additional cost of borrowing $\psi_b D_b^2$. Alternatively, they can be a net recipient of trade credit, in which case they are a debtor.

Two variables in the model reflect the trade credit terms. First, $A_b$ denotes the amount of funds that a buyer, $b$, pays to any seller in period 1. A positive amount of $A_b$ represents a trade credit receipt for the seller or an advance payment for the buyer. Second, $T(z)$ denotes the amount of funds that the seller obtains from a buyer in period 2, where $z$ denotes the realization of the price of the final good. For simplicity, assume there are two states of nature denoted by $\bar{z} > \underline{z} > 0$. The combination of $A_b$ and $T(z)$ determines whether the seller is a net debtor or a net creditor vis-à-vis the buyer. The higher the value for $T(z)$, the more trade credit is awarded to the buyer, since they are more relieved from paying for inputs in advance (the lower is $A_b$) and therefore they are more likely to be a net debtor. Consider the extreme scenario in which $A_b = 0$. Then, the buyer does not pay anything in advance and settles all accounts in period 2. In this case, the buyer is a debtor and the seller is a creditor. Alternatively, suppose $T(z) = 0 \; \forall z$. In this case, the buyer pays the seller entirely in advance, which makes them the creditor and the seller the debtor. Notice that, unlike debt, trade credit is state-contingent. Hence, when the price of the final good is lower, the buyer may give a lower transfer $T(z)$ to the seller in period 2. Thus, the seller can effectively provide insurance to the buyer in bad states and demand compensation in good states via higher transfers $T(z)$. We assume that insurance provision is costly; that is, a seller incurs a cost of $\xi > 0$ units whenever $T(\bar{z}) \neq T(\underline{z})$.

Buyers (sellers) are differentiated by their cost of borrowing, $\psi_b > 0$ ($\psi_s > 0$). Each buyer (seller) draws $\psi_b$ ($\psi_s$) from a uniform distribution with support $[\underline{\psi_b}, \bar{\psi}_b]$ ($[\underline{\psi_s}, \bar{\psi}_s]$) at the beginning of period 1. Buyers and sellers randomly match in period 1 according to the following matching function $M(B,S) = \alpha \frac{B \cdot S}{B+S}$, where $B = 1$ is the measure of buyers, $S = 1$ is the measure of sellers and $\alpha \in (0,2)$ is a parameter that measures the matching efficiency. Hence, the total number of exogenous matches per period is $\frac{\alpha}{2}$. Unmatched agents interact with an intermediary. In particular, unmatched buyers buy the intermediate good from an intermediary at a given price $p_x$. The intermediary does not extend trade credit; hence, debt is the only source of funding for unmatched buyers. Similarly, unmatched sellers sell their (intermediate) product to the intermediary at price $p_x$. The intermediary does not extend them trade credit, so they need to finance all costs of production using debt. In contrast, buyers and sellers who have been matched bargain over the amount of intermediate good produced, $X$, the amount of debt raised in period 1, $D_b$ and $D_s$, the amount of savings between the two periods, $B_b$ and $B_s$, and the terms of trade credit, $A_b$ and $T(z)$.

## 3.1 Dealings with Intermediary

### 3.1.1 Seller's Problem

An unmatched seller has to raise debt from the bank in period 1 in order to produce. They get paid from the intermediary in period 2 when they sell the product to them. They then pay off their debt. The seller's problem is summarized below[2]:

$$\max_{D_s \geq 0, L \geq 1} D_s - \psi_s D_s^2 - wL + \beta \left[p_x \ln L - D_s \left(1 + r^*\right)\right]$$

[2] It is understood that all quantities are specific to a particular seller; for example $D_s(\psi_s)$, where $\psi_s$ identifies a given seller. For ease of exposition, we suppress the notation in this section because we outline the problem for a given individual. We account for the individual's identity when we define equilibrium.

subject to:

$$\begin{aligned} D_s - \psi_s D_s^2 - wL &\geq 0 \\ p_x \ln L - D_s (1 + r^*) &\geq 0 \end{aligned}$$

The first constraint ensures that debt covers the cost of production and borrowing in the first period, while the second guarantees that the proceeds from a sale to the intermediary are enough to cover debt repayment in the second period. The first constraint always holds with equality as it is sub-optimal to waste resources. Using the constraint to substitute out the expression for labor in the objective function and taking FOCs yields the following solution for the optimal amount of debt[3]:

$$D_s = \begin{cases} \frac{2\psi_s + \frac{1+r^*}{p_x} - \sqrt{4\psi_s^2 + \left(\frac{1+r^*}{p_x}\right)^2}}{2\psi_s \frac{1+r^*}{p_x}} & \text{if } p_x \geq (1+r^*)\sqrt{\frac{2\psi_s}{w(2\psi_2 w - 1)}} \\ 0 & \text{if } 0 < p_x < (1+r^*)\sqrt{\frac{2\psi_s}{w(2\psi_2 w - 1)}} \end{cases}$$

Substituting the optimal debt into the first (binding) constraint characterizes labor and production, where $X_s = \ln L$ denotes production by the seller. Substituting optimal debt and labor into the objective function yields the maximized value of a seller with cost draw $\psi_s$, which we denote by $\Gamma_s(\psi_s)$.[4]

### 3.1.2 Buyer's Problem

An unmatched buyer has to raise debt from the bank in the first period in order to buy the intermediate good ($X_b$) from the intermediary. They sell the final good to the consumer in the second period and pay off their debt. The buyer's problem therefore is given by:

[3] See Appendix B for derivation.

[4] It is understood that $\Gamma_s(\psi_s)$ also depends on $p_x$ as well as parameters. We suppress the notation for ease of exposition until we define equilibrium.

$$\max_{D_b, X_b \geq 0} D_b - p_x X_b - \psi_b D_b^2 + \beta E_z \left[ z X_b - D_b \left(1 + r^*\right)\right]$$

subject to:

$$\begin{aligned} D_b - p_x X_b - \psi_b D_b^2 &\geq 0 \\ \underline{z} X_b - D_b \left(1 + r^*\right) &\geq 0 \end{aligned}$$

The two constraints have the same interpretation as in the case of the seller above, with the exception that the second constraint may only be binding in the low state of the world, making the constraint in the high state of the world redundant. The FOCs for the buyer's problem yield the following solution:

$$D_b = \begin{cases} \frac{1}{2\psi_b}\left[1 - \frac{p_x(1+r^*)}{\tilde{z}}\right] & \text{if } 0 < p_x \leq \frac{\underline{z}}{(2-\frac{\underline{z}}{\tilde{z}})(1+r^*)} \\ \frac{1}{\psi_b}\left[1 - \frac{p_x(1+r^*)}{\underline{z}}\right] & \text{if } \frac{\underline{z}}{(2-\frac{\underline{z}}{\tilde{z}})(1+r^*)} < p_x \leq \frac{\underline{z}}{1+r^*} \end{cases}$$

The optimal quantity of intermediate good purchased, $X_b$, follows from the constraints. Substituting optimal debt and intermediate good purchased into the objective function yields the maximized value of a buyer with cost $\psi_b$, denoted by $\Gamma_b(\psi_b)$. This value is decreasing in $\psi_b$, which can be verified by substituting the optimal debt in the objective function.

### 3.1.3 Summary of Unmatched Agents

The buyer and the seller put opposing pressures on the equilibrium price offered by the intermediary. We assume that the intermediary market is perfectly competitive, and that intermediaries earn zero profits by ensuring that the amount of intermediate good purchased equals the amount of intermediate good sold among unmatched agents. We define equilibrium in Section 3.3. Before we do so, however, we turn to the problem that matched agents solve. In that decentralized marketplace, the maximized value functions of the buyer and of the seller from the centralized marketplace, $\Gamma_b(\psi_b)$ and $\Gamma_s(\psi_s)$, respectively, represent outside options. Since $\Gamma_b(\psi_b)$ is decreasing, more debt-constrained buyers have lower outside options and therefore lower bargaining power. Their outside option will play a critical role in the bilateral problem solution, which we describe next.

## 3.2 Bilateral Matches

For simplicity, we assume that sellers make take-it-or-leave-it offers to buyers. The seller solves the following problem:

$$\max_{D_s \geq 0, D_b \geq 0, L \geq 1, \underline{T}, \bar{T}, A_b, B_s, B_b} D_s + A_b - wL - \xi \mathbf{1}\left\{\frac{\bar{T}}{\underline{T}} \neq 1\right\} - \psi_s (D_s)^2 + \beta\left[(B_s - D_s)(1+r^*) + \tilde{T}\right] - \Gamma_s(\psi_s)$$

subject to:

$$D_b - \psi_b (D_b)^2 - A_b + \beta\left[(B_b - D_b)(1+r^*) - \tilde{T} + \tilde{z}\ln L\right] - \Gamma_b(\psi_b) \geq 0 \quad (1)$$

$$B_b = D_b - A_b - \psi_b (D_b)^2 \quad (2)$$

$$B_s = D_s + A_b - wL - \xi \mathbf{1}\left\{\frac{\bar{T}}{\underline{T}} \neq 1\right\} - \psi_s (D_s)^2 \quad (3)$$

$$B_s \geq 0 \quad (4)$$

$$B_b \geq 0 \quad (5)$$

$$(B_s - D_s)(1+r^*) + \underline{T} \geq 0 \quad (6)$$

$$(B_b - D_b)(1+r^*) - \underline{T} + \underline{z}\ln L \geq 0 \quad (7)$$

$$(B_b - D_b)(1+r^*) - \bar{T} + \bar{z}\ln L \geq 0 \quad (8)$$

In the above problem, $\bar{T} \equiv T(\bar{z}), \underline{T} \equiv T(\underline{z})$, and $\tilde{T} \equiv p\underline{T} + (1-p)\bar{T}$, where $p \in (0,1)$ is the probability that the final good price in period 2 is $\underline{z}$; i.e. the probability that the bad state of the world occurs. Furthermore, expression (2) represents the savings of the buyer between the two periods, while (3) denotes the savings of the seller.

The seller maximizes her surplus from being matched over not being matched subject to the constraint in expression (1) that the buyer's surplus within a match does not fall short of their outside option, the constraints in expressions (4) and (5) that savings are non-negative, and the debt repayment constraints for the seller and the buyer, respectively, in expressions (6), (7) and (8). Clearly, if constraint (6) binds when $z = \underline{z}$, it cannot hold with equality when $z = \bar{z}$. Hence, it is sufficient to only consider the former case for the seller. Furthermore, substituting expression (2) into constraint (1) yields

$$\beta \left[ (B_b - D_b)(1 + r^*) - \tilde{T} + \tilde{z} \ln L \right] - \Gamma_b(\psi_b) \geq 0 \tag{9}$$

From expression (9) it follows that constraints (7) and (8) cannot jointly bind for as long as the buyer's outside option is strictly positive. This observation will play an important role in arriving at the solution to the problem. In particular, we will examine various combinations of binding constraints, subject to the restrictions discussed above.

Solving the problem involves characterizing the solutions to four distinct cases. In the first case, the debt repayment constraints (6)-(8) are not binding. This is the unconstrained solution. In the second case, the debt repayment constraint for the seller is binding and debt repayment constraint (7) for the buyer is also binding, while the second-period transfers are equalized across the states of nature in order to avoid paying the insurance cost. In these two cases, trade credit does not provide agents with insurance. In the third (fourth) case, the debt repayment constraint for the seller is binding and debt repayment constraint 7 (8) for the buyer is also binding, but the second-period transfers are not equalized across the states of nature, so the seller incurs the cost $\xi > 0$. These two are the more interesting cases since agents provide each other with insurance via trade credit. We characterize each of these cases in turn below.

Finally, we assume that $\beta(1 + r^*) = 1$. Consequently, agents have no incentives to save, which implies that $B_s = B_b = 0$ and expressions (2)-(5) hold with equality throughout. For tractability, to each allocation below, we assign a numerical subscript that corresponds to the respective case, ex. $D_{b,1}$ is the debt allocation for a given buyer in case 1.

### 3.2.1 Case 1: Unconstrained agents

We assume that the debt repayment constraints (6)-(8) do not bind in any state of the world. This implies that $\bar{T}_1 = \underline{T_1}$ is feasible. Since $\xi > 0$, it is optimal to set $T_1 = \bar{T}_1 = \underline{T_1}$. That is, since insurance is costly to provide, in the equilibrium where agents are not credit constrained and therefore not in need of insurance, trade credit is equalized across states of the world. In addition, the seller always has the incentive to extract all the surplus from the buyer, which implies that constraint (1) is binding. In order to understand how debt levels and production behave in this equilibrium, denote by $\lambda_{i,1}$ the multiplier for the constraint in expression $(i)$ above for case 1, take FOCs, and simplify to obtain:

$$L_1 = \frac{\tilde{z}\beta}{(1 - \tilde{\lambda}_1)w} \tag{10}$$

$$D_{s,1} = \frac{\tilde{\lambda}_1}{2\psi_s\left(\tilde{\lambda}_1 - 1\right)} \tag{11}$$

$$D_{b,1} = \frac{\tilde{\lambda}_1}{2\psi_b\left(\tilde{\lambda}_1 - 1\right)}, \tag{12}$$

where $\tilde{\lambda}_1 \equiv \lambda_{3,1}$. Expressions (11) and (12) imply that, in order to maximize production, each agent will borrow according to her borrowing capacity relative to her partner: $D_s/D_b = \psi_b/\psi_s$. Using this equality, together with expression (10) into constraints (2) and (3) yields a unique solution for the multiplier $\tilde{\lambda}_1 < 1$ given by:

$$\tilde{\lambda}_1 = 1 - \frac{\psi_s \psi_b}{\psi_s + \psi_b} \left[ 2\left(\tilde{z}\beta\right) + \sqrt{\left(\frac{\psi_s + \psi_b}{\psi_s \psi_b}\right)^2 + 4\left(\tilde{z}\beta\right)^2} \right] \quad \text{if} \tag{13}$$

$$\frac{2\underline{z}\beta(\psi_b\psi_s)^2}{(\psi_s+\psi_b)^2} \ln \left( \frac{\frac{\tilde{z}\beta(\psi_s+\psi_b)}{w\psi_s\psi_b}}{\left[ 2\left(\tilde{z}\beta\right) + \sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\tilde{z}\beta\right)^2} \right]} \right) \geq \frac{\psi_s\psi_b}{\psi_s+\psi_b} - \frac{1}{\left[ 2\left(\tilde{z}\beta\right) + \sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\tilde{z}\beta\right)^2} \right]}$$

Having found the optimal $\tilde{\lambda}_1$, we can substitute expression (13) into expressions (10), (11), (12) and constraint (1) which holds with equality, in order to obtain the optimal levels of $L_1, D_{s,1}, D_{b,1}$ and $T_1$, respectively.

### 3.2.2 Case 2: Constrained agents and no insurance

When debt repayment constraints begin to bind, the optimal level of trade credit $T_1$ above is no longer feasible. Since insurance is costly ($\xi > 0$), debt constrained sellers may still find it optimal to not provide insurance to buyers. In this case, $T_2 = \bar{T}_2 = \underline{T_2}$, where $T_2$ is the transfer under the constrained scenario. Further, as discussed above, constraints (7) and (8) cannot be jointly binding. We consider the case where constraint (7) is the binding one.[5] Substituting expression (3) into the objective function for the seller obtains

$$\beta\left[\left(B_{s,2} - D_{s,2}\right)\left(1 + r^*\right) + T_2\right] - \Gamma_s\left(\psi_s\right)$$

Clearly, when $T_2 = \bar{T}_2 = \underline{T_2}$, constraint (6) also cannot bind for as long as the seller's outside option is strictly positive. Similarly, when $T_2 = \bar{T}_2 = \underline{T_2}$ and constraint (7) is binding, constraint (1) cannot be binding as long as the buyer's outside option is strictly positive,

[5] In Appendix B, we demonstrate that a case in which the debt repayment constraint for the seller is binding and debt repayment constraint (8) for the buyer is also binding, but the second-period transfers are equalized across the states of nature in order to avoid paying the insurance cost, is not feasible.

which is apparent from expression (9).

If $\lambda_{i,2}$ is the multiplier for the constraint in expression $(i)$ above, the above discussion implies that $\lambda_{8,2} = \lambda_{6,2} = \lambda_{1,2} = 0$. Taking FOCs and simplifying yields:

$$L_2 = \frac{\underline{z}\beta}{(1-\tilde{\lambda}_2)w} \tag{14}$$

$$D_{s,2} = \frac{\tilde{\lambda}_2}{2\psi_s\left(\tilde{\lambda}_2 - 1\right)} \tag{15}$$

$$D_{b,2} = \frac{\tilde{\lambda}_2}{2\psi_b\left(\tilde{\lambda}_2 - 1\right)}, \tag{16}$$

where $\tilde{\lambda}_2 \equiv \lambda_{3,2}$.

Even when agents are constrained, in order to maximize production, each agent will borrow according to her borrowing capacity relative to her partner, which can be seen from expressions (15) and (16): $D_{s,2}/D_{b,2} = \psi_b/\psi_s$. Using this equality, together with expression (14) into constraints (2) and (3) allows to obtain a unique solution for the multiplier $\tilde{\lambda}_2 < 1$ given by:

$$\tilde{\lambda}_2 = 1 - \frac{\psi_s\psi_b}{\psi_s+\psi_b}\left[2\left(\underline{z}\beta\right) + \sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\underline{z}\beta\right)^2}\right] \quad \text{if} \tag{17}$$

$$\underline{z}\beta \geq \frac{w\psi_s\psi_b}{\psi_s+\psi_b}\left[2\left(\underline{z}\beta\right) + \sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\underline{z}\beta\right)^2}\right]$$

Having found the optimal $\tilde{\lambda}_2$, we can substitute expression (17) into expressions (14), (15), (16) and constraint (7) which holds with equality (imposing that $T_2 = \bar{T}_2 = \underline{T_2}$), in order to obtain the optimal levels of $L_2, D_{s,2}, D_{b,2}$ and $T_2$, respectively.

Notice that, since $\tilde{z} > \underline{z}$, it must be that $\tilde{\lambda}_2 > \tilde{\lambda}$. Since debt is decreasing in the multiplier, $D_{s,2} < D_{s,1}$ and $D_{b,2} < D_{b,1}$. Substituting expression (17) into (14) and taking the derivative with respect to $\underline{z}$ demonstrates that labor is increasing in the price of the final good, so

$L_2 < L_1$. Thus, when the agents are constrained, production as well as debt are lower.

### 3.2.3 Case 3: Constrained agents and insurance; buyer is constrained in bad state

The next two cases are more interesting from the point of view of this paper because they describe situations in which transfers are state contingent, and therefore, trade credit provides agents with insurance. Denote by $\lambda_{i,3}$ the multiplier for the constraint in expression $(i)$ above and combine the FOCs for $\bar{T}_3$ and $\underline{T_3}$ to obtain:

$$\lambda_{8,3}\frac{p}{1-p} = \lambda_{7,3} - \lambda_{6,3}$$

As discussed above, constraints (7) and (8) cannot be jointly binding. We consider the case where constraint (7) is the binding one. Then, $\lambda_{8,3} = 0$ implies that $\lambda_{6,3} = \lambda_{7,3}$.

After combining the FOCs with the constraints and simplifying, we have the following system of eight equations and eight unknowns ($\lambda_{2,3}, \lambda_{4,3}, \lambda_{6,3}, L_3, D_{s,3}, D_{b,3}, \bar{T}_3$ and $\underline{T_3}$):

$$L_3 = \frac{\beta\tilde{z} + \underline{z}\lambda_{6,3}}{(1-\lambda_{2,3})w}$$

$$0 = 1 + \lambda_{2,3} + \lambda_{4,3} + \frac{\lambda_{6,3}}{\beta}$$

$$D_{s,3} = \frac{1+\lambda_{4,3}}{2\psi_{s,3}\left(1-\lambda_{2,3}\right)}$$

$$D_{b,3} = \frac{1+\lambda_{4,3}}{2\psi_{b,3}\left(1-\lambda_{2,3}\right)}$$

$$\frac{\Gamma_b(\psi_b)}{\beta} = \tilde{z}\ln L_3 - p\underline{T_3} - (1-p)\bar{T}_3 - D_{b,3}(1+r^*) \tag{18}$$

$$0 = -\left(D_{s,3} + D_{b,3}\right) + wL_3 + \psi_s\left(D_{s,3}\right)^2 + \psi_b\left(D_{b,3}\right)^2 + \xi \tag{19}$$

$$\underline{T_3} = D_{s,3}(1+r^*) \tag{20}$$

$$D_{b,3} + D_{s,3} = \frac{\underline{z}\ln L_3}{1+r^*} \tag{21}$$

Similarly to the cases above, each agent will borrow according to her borrowing capacity relative to her partner: $D_{s,3}/D_{b,3} = \psi_b/\psi_s$. Using this equality in expression (19) allows us to characterize the optimal level of debt for the seller, $D_{s,3}$. Having found that, $D_{b,3}$ follows trivially from the proportionality result, and $\bar{T}_3, \underline{T_3}$ and $L_3$ follow from expressions (18), (20) and (21), respectively.

From expression (19), the optimal level of the seller's debt solves the following implicit function:

$$(1-\psi_s D_{s,3})\left(1+\frac{\psi_s}{\psi_b}\right) D_{s,3} = we^{\frac{(1+r^*)\left(1+\frac{\psi_s}{\psi_b}\right)D_{s,3}}{\underline{z}}} + \xi \qquad (22)$$

$$\text{if} \quad \frac{1}{4}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right) \geq we^{\frac{1+r^*}{2\underline{z}}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)} + \xi$$

#### 3.2.4 Case 4: Constrained agents and insurance; buyer is constrained in good state

As discussed above, constraints (7) and (8) cannot be jointly binding. Now, we consider the case where constraint (8) is the binding constraint for the buyer, while constraint (6) continues to bind for the seller. Taking FOCs and simplifying yields the following two expressions for debt levels:

$$D_{s,4} = \frac{p\,(1-\lambda_{1,4}) - \lambda_{3,4}}{2\psi_s\,(1-\lambda_{3,4})} \qquad (23)$$

$$D_{b,4} = \frac{p\,(1-\lambda_{1,4}) - \lambda_{3,4}}{2\psi_b\,(1-\lambda_{3,4})} \qquad (24)$$

Constraints (6) and (8) yield the values for $\underline{T_4}$ and $\bar{T}_4$, respectively, which together yield:

$$\tilde{T}_4 = D_{b,4}(1+r^*)\left(p\frac{\psi_s+\psi_b}{\psi_s} - 1\right) + (1-p)\,\bar{z}\ln L_4 \qquad (25)$$

Note that constraint (1) is binding in this case. In fact, because constraint (7) is not binding, we can increase $\underline{T}_4$. This would relax constraint (6) and not affect constraint (8). This change clearly benefits the seller, so the only reason not to do this is if it violates another constraint. That would be constraint (1). Combining constraints (1) and (2) and substituting expression (51) yields an expression for the wage bill:

$$wL_4 = we^{\frac{D_{b,4}(1+r^*)p\left(1+\frac{\psi_b}{\psi_s}\right)+\frac{\Gamma_b(\psi_b)}{\beta}}{p\underline{z}}} \tag{26}$$

Similarly to previous cases, each agent will borrow according to her borrowing capacity relative to her partner: $D_{s,4}/D_{b,4} = \psi_b/\psi_s$. Using this equality together with expression (26) in constraints (2) and (3) allows us to characterize the optimal level of debt for the seller, $D_{s,4}$. Having found that, $D_{b,4}$ follows trivially from the proportionality result, and $\underline{T}_4, \bar{T}_4$ and $L_4$ follow from constraints (6) and (8) and expression (26), respectively. The optimal level of debt $D_{s,4}$ solves the following implicit function:

$$(1-\psi_s D_{s,4})\left(1+\frac{\psi_s}{\psi_b}\right)D_{s,4} = we^{\frac{(1+r^*)\left(1+\frac{\psi_s}{\psi_b}\right)D_{s,4}}{\underline{z}}+\frac{\Gamma_b(\psi_b)}{\beta p\underline{z}}} + \xi \tag{27}$$

$$\text{if} \quad \frac{1}{4}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right) \geq we^{\frac{1+r^*}{2\underline{z}}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)+\frac{\Gamma_b(\psi_b)}{\beta p\underline{z}}} + \xi$$

The solutions to cases 3 and 4 closely resemble each other. They both yield state contingent transfers. The level of the transfers, as well as the level of debt and production varies across the cases. In Appendix B, we show that case 4 yields lower levels of debt and production than case 3.

## 3.3 Equilibrium

Let $\Omega \equiv \{w, \beta, r^*, p, \underline{z}, \bar{z}, \xi\}$ be a vector of parameters in the model. Let $V_{s,j}(\psi_s, \psi_b; p_x, \Omega)$ denote the value of a seller with cost draw $\psi_s$ who is matched to a buyer with cost draw

$\psi_b$ in case $j \in \{1,2,3,4\}$.[6] Similarly, let $V_{b,j}(\psi_s, \psi_b; p_x, \Omega)$ denote the value of a buyer with cost draw $\psi_b$ who is matched to a seller with cost draw $\psi_s$ in case $j \in \{1,2,3,4\}$. Define the indicator $I_s(\psi_s, \psi_b; p_x, \Omega)$ to be 1 when $\forall j \in \{1,2,3,4\}$, $\Gamma_s(\psi_s; p_x, \Omega) \geq V_{s,j}(\psi_s, \psi_b; p_x, \Omega)$. Recall that the probability of a match (unconditional on bargaining outcomes) is $\frac{\alpha}{2}$, which implies that $1 - \frac{\alpha}{2}$ is the probability that agents will not be given the opportunity to match. Then, aggregate supply of the intermediate good is:

$$S(p_x, \Omega) = \left(1 - \frac{\alpha}{2}\right) \int_{\underline{\psi_s}}^{\bar{\psi_s}} X_s\left(\psi_s; p_x, \Omega\right) d\psi_s + \frac{\alpha}{2} \int_{\underline{\psi_s}}^{\bar{\psi_s}} \int_{\underline{\psi_b}}^{\bar{\psi_b}} I_s(\psi_s, \psi_b; p_x, \Omega) X_s\left(\psi_s; p_x, \Omega\right) d\psi_b d\psi_s \quad (28)$$

Define the indicator $I_b(\psi_s, \psi_b, ; p_x, \Omega)$ to be 1 when $\forall j \in \{1,2,3,4\}$, $\Gamma_b(\psi_b; p_x, \Omega) \geq V_{b,j}(\psi_s, \psi_b; p_x, \Omega)$. Then, aggregate demand for the intermediate good is:

$$D(p_x, \Omega) = \left(1 - \frac{\alpha}{2}\right) \int_{\underline{\psi_b}}^{\bar{\psi_b}} X_b\left(\psi_b; p_x, \Omega\right) d\psi_d + \frac{\alpha}{2} \int_{\underline{\psi_s}}^{\bar{\psi_s}} \int_{\underline{\psi_b}}^{\bar{\psi_b}} I_b(\psi_s, \psi_b; p_x, \Omega) X_b\left(\psi_b; p_x, \Omega\right) d\psi_b d\psi_s \quad (29)$$

Market clearing in the centralized market implies that

$$S(p_x, \Omega) = D(p_x, \Omega) \quad (30)$$

**Definition 1** *For given parameter set* $\Omega$*, equilibrium is a price* $p_x \in (0, \infty)$*, allocations for the centralized market* $\{D_s(\psi_s; p_x, \Omega), D_b(\psi_b; p_x, \Omega), X_s(\psi_s; p_x, \Omega), X_b(\psi_b; p_x, \Omega)\}$*, Lagrange multipliers for the decentralized market* $\{\tilde{\lambda}_1(\psi_s, \psi_b; p_x, \Omega), \tilde{\lambda}_2(\psi_s, \psi_b; p_x, \Omega)\}$*, and debt allocations for the decentralized market* $\{D_{s,3}(\psi_s, \psi_b; p_x, \Omega), D_{b,3}(\psi_s, \psi_b; p_x, \Omega), D_{s,4}(\psi_s, \psi_b; p_x, \Omega),$ $D_{b,4}(\psi_s, \psi_b; p_x, \Omega)\}$ *that satisfy: (i) the buyer's problem's solution in the centralized market, (ii) the seller's problem's solution in the centralized market, (iii) market clearing in the centralized market given by expression (30), (iv) shadow prices in expressions (13) and (17); (v) the optimal levels of seller debt in the decentralized markets given by expressions (53)*

[6]The exact expression for each case can be found in Appendix B.

*and (27); and (vi) the optimal levels of buyer debt in the decentralized markets that satisfy* $D_{b,j}(\psi_s, \psi_b; p_x, \Omega) = D_{s,j}(\psi_s, \psi_b; p_x, \Omega)\frac{\psi_s}{\psi_b}$ *for* $j = \{3, 4\}$.

### 3.4 Domestic-Currency and Foreign-Currency Debt

In the model that we develop above, the interest rate $r^*$ is exogenous. Implicitly, we assume that the country is a small open economy where agents borrow from the world market. Further, all debt is denominated in units of real output.

In Appendix A, we extend the model to feature a portfolio choice problem for each agent who can raise debt denominated in domestic currency and in foreign currency as well as obtain trade credit. In particular, for each agent $j = \{b, s\}$, we let $D_j$ denote the value of domestic-currency debt, expressed in terms of domestic output, and $D_j^I$ denote the value of foreign-currency debt, expressed in domestic output. All debt incurs the interest rate $r^*$. However, we allow the (quadratic) cost of debt to differ across the two denominations. In particular, we assume that, at the beginning of the first period, each agent $j$ draws a realization of $(\psi_j^I, \psi_j)$ from a joint distribution, where $\psi_j^I$ is the cost of borrowing in foreign currency. We let moments from the cross-firm distribution of foreign- and domestic-currency debt inform us about the parameters of the joint distribution of costs. In particular, a positive correlation between $\psi_j^I$ and $\psi_j$ implies that firms that can borrow cheaply in home currency can also borrow cheaply in foreign currency. Additionally, if the mean value of $\psi_j^I$ across all agents is below the mean value of $\psi_j$, then foreign-currency debt is on average cheaper than domestic-currency debt. These predictions are in line with observations for Hungary, as reported by Salomao and Varela (2018). Finally, we assume that agents incur a fixed cost out of their profits if they decide to borrow in foreign currency, $f^I > 0$. The latter assumption ensures that only the largest firms access foreign-currency debt markets, as documented by Salomao and Varela (2018).

The generalized model predicts that each agent who chooses to access the foreign-currency debt market raises debt in foreign currency that is in proportion to their domestic-currency borrowing, where the proportionality factor is given by the ratio of borrowing costs, $\frac{D_j^I}{D_j} = \frac{\psi_j}{\psi_j^I}$, for $j = \{b, s\}$. Furthermore, the model's solution features four distinct cases, as described above, where the optimal allocations resemble the allocations in the baseline model. Finally, the predictions that we derive below regarding the behavior of trade credit relative to total debt (domestic- and foreign-currency) remain unchanged. Since the ORBIS database, which we use to validate the model's predictions regarding trade credit, does not include details on the currency denomination of debt, we focus on the simple model in the main text, and we relegate all discussions of the dual-currency model to Appendix A.

# 4 Numerical Analysis: Model Predictions

To better highlight the insights from the model we perform a brief numerical exploration. We set the matching efficiency to 1.6 such that 80% of sellers are offered the possibility of matching with a buyer. The exogenous wage is set at 0.1. With probability 25% the buyer faces a low idiosyncratic demand of 2 and with 75% probability the demand is 5. The discount factor is set to 0.95 and the fixed cost of providing insurance to the buyer using contingents transfers is set to 0.25. Table 2 shows the parameter values used for the numerical illustration of the model.

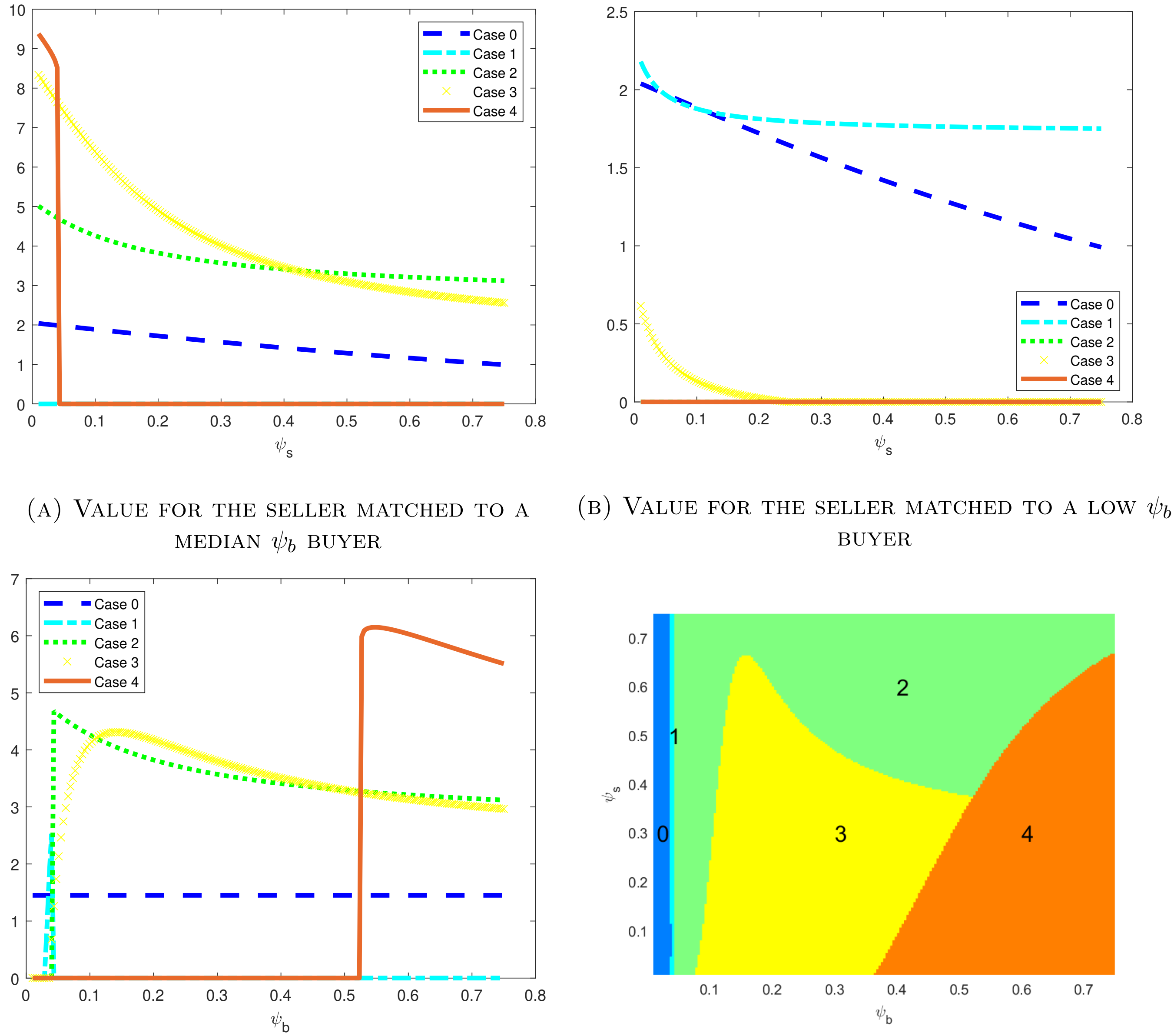
We solve the model for a uniform distribution of sellers (buyers) with 200 types such that $\psi_i \in [0.01, 0.75]$. The equilibrium price of the intermediate good when dealing with the broker is 1.4.

| Parameter | Description | Value |
|---|---|---|
| $\alpha$ | Matching efficiency | 1.6 |
| $w$ | Labor cost | 0.1 |
| $\underline{z}$ | Low demand | 2 |
| $\bar{z}$ | High demand | 5 |
| $p$ | Probability of low demand | 0.25 |
| $\beta$ | Discount factor | 0.95 |
| $\xi$ | Cost of providing contingent $T$ | 0.25 |

Table 2: Parameter values

Figure 4 illustrates the equilibrium choices of a seller offered the opportunity to match with a buyer. The first three panels show for every case (0 to 4, where 0 denotes unmatched agent) the value $V_{s,j}(\psi_s, \psi_b)$ of a seller who was offered the possibility to match with a buyer. Note that, in case 0, the seller chooses to exercise their outside option. When a given case is not feasible, we set the value of that case to zero. Panel 4a displays the value for the seller when matched to the median buyer as a function of the type of the seller. Note that, for every case, the value of the seller decreases in $\psi_s$ as borrowing becomes more expensive for the seller. When facing the median buyer, accepting a match strictly dominates dealing with the broker (case 0 is never preferred). Also, in this type of a match, the unconstrained allocation is not feasible, which is why case 1 always appears to be at value of 0. When the seller has access to relatively cheap bank loans (low $\psi_s$), the best option is to provide insurance to the buyer using contingent trade credit (cases 3 and 4). As borrowing becomes more expensive for the seller the dominating strategy is to provide non-contingent trade credit (case 2). Panel 4b shows the value for the seller when matched with a buyer who has a low value of $\psi_b$. In this case, the buyer does not need insurance from the seller since debt is relatively cheap for them, and the outside option for the buyer (dealing with the broker) is

Figure 4: Seller's Value function and Equilibrium Cases




(A) Value for the seller matched to a median $\psi_b$ buyer

(B) Value for the seller matched to a low $\psi_b$ buyer

(C) Value for the median $\psi_s$ Seller

(D) Case Policy

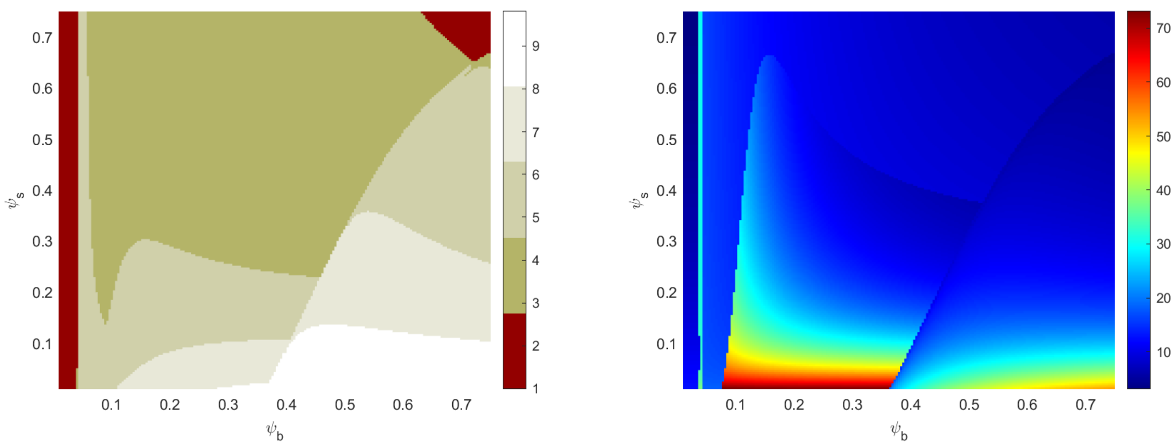
relatively high. Therefore, the seller either decides to refuse the match (case 0) or to select a non-contingent strategy (the unconstrained allocation in this match since it is feasible).

Panel 4c takes a different approach focusing on the median seller and seeing how their value changes as a function of the type of the buyer. Naturally, the value of the seller when interacting with the broker (case 0) is independent of the type of the buyer. Consistent with our former analysis, when the median seller faces a buyer with access to cheap credit, they decide to refuse the match as compensating the buyer for the outside option is very expensive. As the borrowing cost of the buyer increases, the seller decides to match without providing insurance (cases 1 and 2), only when the borrowing cost of the buyer becomes high enough, the seller decides to provide insurance (cases 3 and 4). Note that the value function of the seller is not always monotonic on the type of the buyer. This non-monotonicity is particularly clear for case 3. On the one hand, as the borrowing cost of the buyer increases, their outside option decreases allowing the seller to capture more surplus. On the other hand, when the borrowing cost of the buyer is too expensive, the joint borrowing capacity of the match is compromised and the production scale decreases, implying a lower surplus for the pair.

Panel 4d maps the complete equilibrium space. In a nutshell, sellers matched with. buyers who have very low $\psi_b$ prefer to deal with the broker. When the buyer can borrow at low cost, or when borrowing is very expensive for both the buyer and the seller, the deal features non-contingent transfers. For buyers with higher cost $\psi_b$, the seller provides at least some insurance. In this equilibrium, 3.6% of the sellers refuse the match offered (case 0) and prefer to interact with the broker. Taking into account the agents that were not offered a match, 22.8% of the buyers and sellers interact with the broker. Only 0.8% of the sellers end up in the unconstrained case (case 1), 29.4% are in case 2 and offer non-contingent trade credit, while case 3 and case 4 occur for 24.7% and 22.3% of the sellers, respectively. Therefore, almost 50% of the buyers are allowed to deliver contingent transfers to pay for their inputs.

Figure 5 shows how the value of the seller and their hiring decision are determined by the cost of borrowing of the match. Panel 5a show that sellers with low borrowing cost who are matched to buyers with high cost benefit the most from being matched (white region). Interestingly, these pairs are not the ones with the highest production. In fact, Panel 5b shows that low-$\psi_s$ sellers matched with low- to medium-$\psi_b$ buyers are the ones hiring more workers and therefore producing more intermediate goods. These sellers cannot benefit as much from the borrowing needs of the buyer and instead focus on maximizing the total surplus increasing production. Moreover, even relatively high-$\psi_s$ sellers are able to scale production significantly when matched with low $\psi_b$ buyers under a contingent transfer scheme (left border between cases 2 and 3 in Figure 4d).

Figure 5: Value and Labor

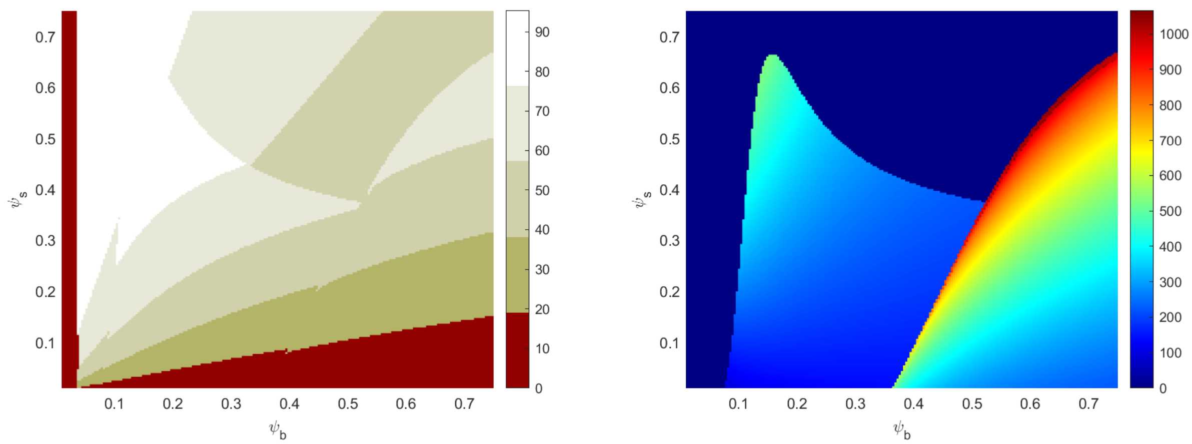



(a) Value for the Seller

(b) Labor Hired by Seller

The contrast between value and production in Figure 5 highlights the duality of trade credit. First, trade credit allows a high-$\psi_s$ seller ($\psi_b$ buyer) to scale up production by taking advantage of matching with a low-$\psi_b$ buyer ($\psi_s$ seller). The goal of trade credit in this case is to find cheap finance for intermediate good production. A second goal for trade credit is to provide repayment insurance to the buyer. This role is particularly important when buyers and sellers face moderate spreads. In that range, the buyer incurs non-trivial borrowing costs and, therefore, the second period repayment constraint is likely to bind in some state. In this situation, contingent trade credit allows the pair to achieve higher production by allowing the buyer to repay in every state.

Figure 6: Trade Credit: Scale and Insurance



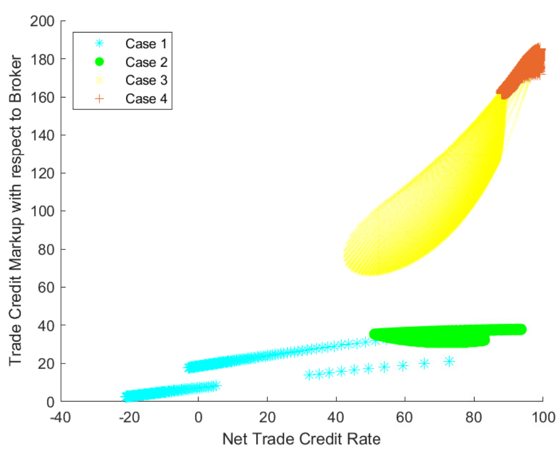

(a) Labor cost covered by Buyer

(b) State Contingency

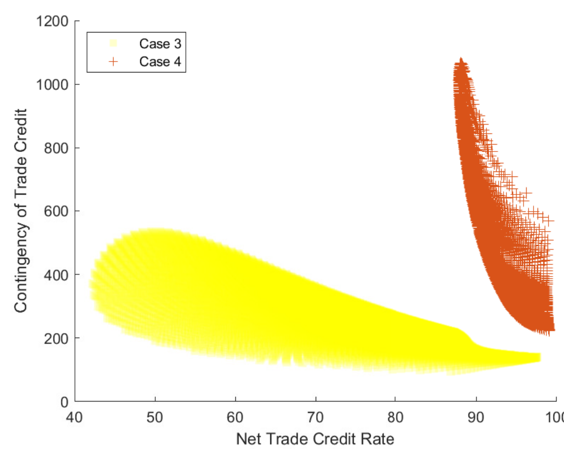
Figure 6 explores the dual role of trade credit in the model. First, Panel 6a reflects the fraction of the labor cost of the seller that is covered by trade credit from the buyer $\left(\frac{A_b}{wL}\right)$ in percentage terms. Lighter colors indicate that the labor cost of the seller is intensively financed by trade credit from the buyer. It is clear that low-$\psi_b$ buyers can also help high-$\psi_s$ sellers achieve a higher production scale. In fact, up to 90% of the production cost of the seller in Panel 6a can be covered with trade credit from the buyer. Panel 6b plots the ratio between the contingent payments agreed upon by the seller and the buyer, in percentage terms, $\left(\frac{\bar{T}}{\underline{T}} - 1\right)$. This figure emphasizes the insurance role of trade credit. Lighter colors signal payments that are highly contingent on the demand realization. Although, with the exception of case 0, every case exhibits some trade credit, only cases 3 and 4 feature contingent trade credit. For a given $\psi_b$, when $\psi_s$ increases, the buyer receives more contingent trade credit. The reason is simple, the more costly borrowing becomes for the seller, the more borrowing is channeled through the buyer, therefore, the more insurance the buyer needs in order to be able to repay in both states of nature. Similarly, for a given $\psi_s$, contingency decreases with $\psi_b$ as the seller borrows more and needs to repay unconditionally to the bank.

Typically trade credit carries a high implicit cost for the borrower (see Klapper et al. (2011)). Figure 7 studies how the cost, size, and contingency of trade credit are related. We infer the cost of trade credit for the buyer by comparing the average unit price of an intermediate good within the match to the market price when interacting with the broker; since a market price is not directly observed in a bilateral meeting, we plot the total values of each exchange, in percentage terms, $\left(\frac{A_b+\beta\tilde{T}}{p_x X} - 1\right)$. This is an effective markup over the centralized market price and reflects how costly trade credit is.

FIGURE 7: THE COST OF TRADE CREDIT




(A) NET TRADE CREDIT TO THE BUYER




(B) LABOR COST COVERED BY BUYER

Panel 7a plots the mark-up against the expected net trade credit rate received by the buyer defined as $\frac{\beta\tilde{T}-A_b}{A_b+\beta\tilde{T}}$ and plotted in percentage terms. The numerator reflects the difference between the discounted expected transfers that the buyer will make after receiving the intermediate inputs and the payment that they make before the seller delivers the intermediate goods. The denominator is the total size of the expected payout that the buyer makes to the seller. Note that, for all cases, the more credit the seller extends to the buyer, the more expensive trade credit becomes. Interestingly, under case 1, the buyer can be a net lender to the seller, while in all other cases the buyer is always a net debtor. Moreover, contingent trade credit (cases 3 and 4) is associated with higher trade credit markups. Panel 7b plots the contingency measure described in Panel 6b against the expected net trade credit rate received by the buyer for the cases that exhibit contingent trade credit. Note that case 4 is characterized by large, highly contingent and extremely expensive trade credit for the buyer, while trade credit under case 3 is less contingent, smaller, and cheaper. Finally, note that, within each case, the more trade credit the seller provides, the less contingent it becomes.

# 5 Empirical Analysis: Test of Model's Predictions

In this section, we examine some predictions of the model using firm-level data from Hungary. We utilize the ORBIS database, which has annual firm-level observations with balance-sheet and other information. We choose to work with Hungary, an emerging economy, because the ORBIS database coverage for this country is exceptionally high.[7] In addition, Salomao and Varela (2018) use Hungarian firm-level data to document stylized facts regarding firm currency choice of debt, albeit they do not study the behavior of trade credit. Therefore, we see our paper as a complement to theirs. While we do not have detailed information on the currency of debt denomination in the ORBIS database, the general model of trade credit with debt denominated in domestic and in foreign currency that we develop is in line with the observations reported in Salomao and Varela (2018).

We clean the sample, dropping unusual observations (ex. negative assets or employees).[8] The main restriction on our sample is coverage for variables of bank debt, trade credit borrowing, and trade credit lending. For these variables, it is difficult to distinguish between a true 0 and a missing observation marked with a 0. We restrict our sample to firms that had

[7] Bajgar et al. (2020) describe the degree of coverage for different countries in the ORBIS database. For robustness, we are currently working on specifications that include multiple countries.

[8] We largely follow the construction/cleaning process outlined in Kalemli-Ozcan et al. (2015). We drop financial and public administration firms from our sample.

non-missing data over 2009-2014, and had a positive value for one of these three variables at least once over that period. This reduces our sample to about 8300 firms over 10 years.

**Testable Prediction 1: Trade credit is driven by debt constraints of revenue-maximizing firms.** We first examine the relationship between bank debt and the level of trade credit borrowing. In the model, a producer's cost of borrowing, as well as the cost relative to the trading partner, determines the amount of trade credit they receive. These costs are not observable in the data; however, according to the model, the costs directly speak to debt levels of agents, which are observable. Focusing on the buyer (the results are identical from the point of view of the seller), we run the following regression in the model:

$$TradeCreditReceived_b(\psi_s, \psi_b) = \alpha + \beta D_b(\psi_s, \psi_b) + \gamma Revenue_b(\psi_s, \psi_b) + \delta \frac{D_b(\psi_s, \psi_b)}{D_s(\psi_s, \psi_b)} + \epsilon_{s,b},$$

where trade credit for the buyer is defined as $\beta\tilde{T}$, revenue of the buyer is $\beta\tilde{z}X$, and debt levels are as in the theory above. We run the regression using every matched pair of $(\psi_s, \psi_b)$ in the equilibrium of the model.

TABLE 3: TRADE CREDIT BORROWING AND DEBT - MODEL

| | (1) | (2) | (3) |
|---|---|---|---|
| Bank Debt Buyer | -*** | -*** | -*** |
| Revenue Buyer | | +*** | +*** |
| Bank Debt Buyer / Bank Debt Seller | | | -*** |
| Observations | 40000 | 40000 | 40000 |
| $R^2$ | 0.1194 | 0.4245 | 0.6629 |

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The regression results are summarized in Table 3. We are interested in the signs of the coefficients rather than their magnitudes. The model predicts that, for any buyer, debt and trade credit are substitutes, which gives a negative coefficient estimate for $\beta$. Conditional on the amount of debt, the larger is the buyer's scale of production, the more trade credit they receive in order to finance it, which yields a negative coefficient estimate of $\gamma$. Finally, the coefficient $\delta$ captures the effect of the debt capacity of the buyer relative to the seller on trade credit received by the buyer. This coefficient is negative because the buyer receives more trade credit the more debt-constrained they are compared to their partner. Notice that these three variables explain 66% of the variation of trade credit in the model.

We run a parallel regression using Hungarian data:

$$
\begin{aligned}
&TradeCreditReceived_{it} = \\
&\alpha_i + \alpha_{st} + \beta BankDebt_{it} + \gamma Revenue_{it} + \delta BankDebt_{it}/SellerBankDebt_{st} + \epsilon_{it},
\end{aligned}
$$

where $i$ denotes a firm, $t$ denotes a year, and $s$ denotes the 2-digit ISIC sector that firm $i$ is in.[9] We control for firm and sector-year fixed effects to account for time invariant differences across firms and shocks to specific industries. All variables are in millions USD and are winsorized at 1%.

While in the model, debt and trade credit received are the only assets of the firm, in the data that is not the case. Consequently, we normalize the variables in the data to account for different types of firm assets. The dependent variable is the share of trade credit (accounts payable) in short-term debt (trade credit + other short-term debt). We opt to use short-term debt as a natural proxy for debt since, in our model, firms use debt to finance daily operations.[10] This interpretation is standard in the existing literature on financial frictions (see Dinlersoz et al. (2019)). The variable $BankDebt_{it}$ is the firm's short-term debt, relative to total assets, while $Revenue_{it}$ is the firm's sales revenue, relative to total assets. $SellerBankDebt_{st}$ is computed using our sample of firms and the input-output table for Hungary from WIOD (see Timmer et al. (2015)). Then, $BankDebt_{it}/SellerBankDebt_{st}$ is

[9] The firms in the sample span 48 2-digit ISIC sectors.

[10] Our results are robust to using total assets in the denominator, which include long-term debt and other assets.

given by the firm's short-term debt relative to the average short-term debt of firms in sectors that the firm's sector buys from and sells to in each year over 2009-2014. The average is computed using weights from industry input-output tables. The input-output table is prepared up through 2014, which limits the analysis to 2009-2014, although ORBIS data spans 2009-2018.[11]

Table 4: Trade credit borrowing and debt - Hungary

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| $\text{Debt}_{it}$ | -1.481*** | -1.514*** | -1.353*** | -1.482*** |
| | (0.0579) | (0.0541) | (0.0488) | (0.0417) |
| $\text{Sales}_{it}$ | | 0.0131*** | 0.00915*** | 0.00728*** |
| | | (0.00201) | (0.00196) | (0.00257) |
| $\text{Debt}_{it}/\text{PartnerDebt}_{st}$ | | | -0.0562*** | -0.0572*** |
| | | | (0.00852) | (0.00781) |
| $\text{EBITDA}_{it}$ | | | | -0.0194 |
| | | | | (0.0128) |
| $\text{Capital}_{it}$ | | | | 0.0149 |
| | | | | (0.0166) |
| Log $\text{Emp}_{it}$ | | | | 0.00356 |
| | | | | (0.00541) |
| Log $\text{Assets}_{it}$ | | | | -0.00819 |
| | | | | (0.00861) |
| Observations | 47297 | 45778 | 29703 | 24867 |
| $R^2$ | 0.384 | 0.393 | 0.392 | 0.421 |
| FirmFE | Yes | Yes | Yes | Yes |
| SectorYearFE | Yes | Yes | Yes | Yes |

Dependent variable is the share of trade credit (accounts payable) in short-term debt (trade credit + other ST debt). Debt is current (short-term) debt liabilities. Sales is the net sales revenues. The latter two are normalized by total assets. Debt ratio is computed as the ratio of the firm's debt to the (weighted) average debt of firms in sectors that the firm's sector buys from and sells to in each year over 2009-2014, with weights computed from input-output tables. Other firm controls include EBITDA over assets, capital over assets, log employment, and log assets. All independent variables (except those in logs) are winsorized at 1%. Sample spans 2009-2014. $R^2$ is within $R^2$. Errors are clustered at the industry level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4 reports the regression results using firm-level data from Hungary. In the first

[11] The results are robust to using 2009-2018 ORBIS data with an input-output matrix that is equal across years and computed using the mean value for the 2009-2014 period.

column, we regress trade credit that a firm receives on its total amount of debt credit, and we find a negative and statically significant coefficient as predicted by theory. In the second column, we add the firm's sales revenue, which yields a positive coefficient estimate, as predicted by theory. Finally, the model predicts that the amount of trade credit exchanged between two firms depends both on the borrower (demand) and lender (supply) characteristics: less debt-constrained firms are net lenders relative to more debt-constrained ones. This type of bilateral measure is not available in our data set at the firm level because we do not know the identify of the trade credit provider/recipient for each firm; rather, we only observe the gross amounts of trade credit issued and received. We construct a proxy for this variable as described above using industry-level data, and we include it in the regression the results of which are reported in the third column. The negative coefficient estimate on the variable that measures the relative debt of a firm to its trading partners in the third row confirms the theoretical prediction. To interpret the results, focusing on the third column, if a firm has twice as much debt as its partners–a ratio of 2–the coefficient estimate of -0.056 implies that the impact on the firm's trade credit borrowing ratio is -0.112, or trade credit is 11.2 percentage points lower, as percent of short-term borrowing.

In the fourth column, we incorporate additional firm-level controls. In particular, we include the firm's EBITDA in order to capture the firm's profitability (see Lian and Ma (2021)). Since high-profit have more retained earnings, they may rely less on other sources of financing, most notably trade credit. In addition, the scale of the firm's operation may be affected by its total amount of capital or assets, so we control for both. Finally, we include firm employment as an additional proxy for firm size. None of these variables appear to be statistically significant, and more importantly, the baseline estimates of the coefficients of interest remain unchanged. Overall, we conclude that the empirical results provide support for the first testable prediction.

**Testable Prediction 2: Trade credit provides insurance along supply chains.** We examine the volatility of trade credit received. Since trade credit is state contingent, the higher is the volatility of trade credit, the more insurance an agent receives because they have to pay less in a bad state of the world. We run the following regression in the model:

$$Vol\left[\frac{TradeCreditReceived_b(\psi_s,\psi_b)}{Revenue_b(\psi_s,\psi_b)}\right] =$$
$$\alpha + \beta D_b(\psi_s,\psi_b) + \gamma\frac{D_b(\psi_s,\psi_b)}{D_s(\psi_s,\psi_b)} + \delta Revenue_b(\psi_s,\psi_b) + \epsilon_{s,b},$$

where the regressand is given by the following expression $\frac{\beta\tilde{T}}{\beta\tilde{z}X}$, which represents the standard deviation of trade credit, relative to revenue, for the buyer computed over the two states of nature in the model.

Table 5: Trade credit borrowing volatility - Model

| | (1) | (2) | (3) |
|---|---|---|---|
| Bank Debt Buyer | -*** | -*** | -*** |
| Revenue Buyer | | -*** | -*** |
| Bank Debt Buyer / Bank Debt Seller | | | +*** |
| Observations | 40000 | 40000 | 40000 |
| $R^2$ | 0.2786 | 0.3722 | 0.4084 |

* p < 0.10, ** p < 0.05, *** p < 0.01

The regression results are summarized in Table 5. More variance implies more insurance, since the buyer pays less as a fraction of sales in a bad state and more in a good state. When the buyer has low $\psi_b$, and hence a lot of debt, they are very likely to repay debt unconditionally and they do not need insurance, which gives a negative coefficient estimate on debt. A high ratio of debt indicates that the buyer is doing most of the borrowing. Suppose that a buyer and a seller need to borrow a total of $100 to finance production, and the buyer carries the majority of debt, 80%. Then, it is more likely that the buyer, relative to the seller, will be constrained in some state of the world, which is why they need relatively more trade credit, conditional on their total debt. This yields a positive coefficient estimate on the ratio of debt, conditional on the buyer's debt (column (2)). Finally, more revenue implies that the buyer will have more funds to repay their debt so they are not in need of insurance, which yields a negative coefficient estimate.

We next examine the volatility of trade credit borrowing in the data. The volatility is measured as the standard deviation of the share of trade credit borrowing in short-term debt (which is the sum of accounts payable and short-term debt) across our sample (2009-2014). Necessarily, we collapse our data to firm-level observations, regressing this standard deviation on the same variables as in the first empirical exercise above. All variables are winsorized at the 1% level after having computed the medians and standard deviation. The regression looks as follows:

$$Vol[TradeCreditReceived_{it}] =$$
$$\alpha_i + \alpha_{st} + \beta BankDebt_{it} + \gamma Revenue_{it} + \delta BankDebt_{it}/SellerBankDebt_{st} + \epsilon_{it}.$$

Table 6 reports the results of the regression using firm-level data from Hungary. Unlike the model's predictions, firms with larger debt receive more insurance, measured by the volatility of trade credit received. This result is not surprising because the value of debt in the data reflects additional information than the firm's cost of borrowing alone. In fact, firms with higher debt levels may be more likely to default, and hence they receive more insurance in bad states of the world. More importantly, similarly to the model's predictions, larger firms, whether measured by their sales revenues or employment, receive less insurance, and firms that are more constrained, relative to their trade partner, receive more insurance. It is this last coefficient estimate that offers direct support of our theory of insurance.

Table 6: Trade credit borrowing volatility - Hungary

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| $\text{Debt}_i$ | 0.00184*** | 0.00204*** | 0.00170*** | 0.00163*** | 0.00173*** |
| | (0.000265) | (0.000250) | (0.000234) | (0.000246) | (0.000254) |
| $\text{Sales}_i$ | | -0.00823*** | -0.00737*** | -0.00816*** | -0.00809*** |
| | | (0.00202) | (0.00197) | (0.00203) | (0.00206) |
| $\text{Debt}_i/\text{PartnerDebt}_s$ | | | 0.0206*** | 0.0212*** | 0.0253*** |
| | | | (0.00287) | (0.00267) | (0.00351) |
| $\text{EBITDA}_i$ | | | | -0.00209 | 0.00491 |
| | | | | (0.0120) | (0.0137) |
| $\text{Capital}_i$ | | | | 0.00615* | 0.00523 |
| | | | | (0.00340) | (0.00459) |
| Log $\text{Emp.}_i$ | | | | | -0.00538*** |
| | | | | | (0.00166) |
| Log $\text{Assets}_i$ | | | | | -0.00166 |
| | | | | | (0.00106) |
| Observations | 6666 | 6610 | 6610 | 6447 | 6103 |
| $R^2$ | 0.0219 | 0.0296 | 0.0358 | 0.0365 | 0.0446 |
| SectorFE | Yes | Yes | Yes | Yes | Yes |

Regression is run at the firm level (one observation per firm). Dependent variable is the standard deviation over the sample for each firm of the share of trade credit borrowing (accounts payable) in short-term debt (trade credit + other ST debt). Debt is sample average of current (short-term) debt liabilities for each firm relative to total assets. Sales is sample average of the net sales revenues to total assets. Debt ratio is computed as the ratio of the firm's debt to the average debt of firms in sectors that the firm's sector buys from and sells to in each year over 2009-2014, with a weighted average across those sectors based on the relative size of their inputs to and outputs from the firm's sector (from industry input-output tables) - the average over the sample for each firm is used. Other firm controls include the sample average for each firm of: EBITDA over assets, capital over assets, log employment, and log assets (the latter two are averaged first, and then logged). All variables (except those in logs) are winsorized at 1%. $R^2$ is within $R^2$. Errors are clustered at the industry level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

## 6 Conclusion

We study the interplay between bank and trade credit. Using data for Mexico, we document that small and medium-sized enterprises (SMEs) trade off bank for trade credit, while large firms are more likely to extend trade credit, especially during financial crises. We develop a model of heterogeneous firms that extend state-contingent credit to each other along supply chains for the purpose of providing insurance in the case of adverse economic shocks. The model predicts that firms obtain more trade credit the less bank credit they have available, the larger is their scale of operation, and the more-debt constrained they are relative to their trading partner. Further, more debt-constrained firms receive more state-contingent trade credit from their partners. We validate the model's predictions using firm-level data from Hungary. We conclude that the insurance channel of trade credit earns it a role of a macroeconomic stabilizer in emerging markets.

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# A General Model of Trade Credit with Domestic- and Foreign-Currency Debt

In this section, we outline a model that includes domestic- and foreign-currency debt alongside trade credit. The model nests the simple model of trade credit that we outline in the main text. We maintain the small-open economy assumption and we allow each agent to borrow at ongoing interest rate $r^*$. Borrowing incurs a quadratic cost, which differs according to the currency of debt issue. Each agent draws $(\psi_j, \psi_j^I), j = b, s$ from a joint distribution, where $\psi_j^I$ is the cost parameter for foreign-currency debt for agent $j$. If they decide to access the foreign-currency market, agents must pay a fixed cost out of their total profits, denoted by $f^I > 0$.

Below we outline the solution to the model following the same structure as in the main text. Once again, agents have the opportunity to transact with an intermediary in a centralized market, and if matched, with a counter party in a decentralized market.

## A.1 Dealings with Intermediary

### A.1.1 Seller's Problem

We denote a seller by their cost draws $(\psi_s, \psi_s^I)$.

**Seller borrows in domestic currency**

This problem is solved in the main text.

**Seller borrows in domestic and foreign currency**

In this case, the seller solves:

$$\max_{D_s \geq 0, D_s^I \geq 0, L \geq 1} D_s + D_s^I - \psi_s D_s^2 - \psi_s^I (D_s^I)^2 - wL + \beta \left[ p_x \ln L - \left( D_s + D_s^I \right) (1 + r^*) \right] - f^I$$

subject to:

$$\begin{aligned} D_s + D_s^I - \psi_s D_s^2 - \psi_s^I (D_s^I)^2 - wL &\geq 0 \\ p_x \ln L - \left( D_s + D_s^I \right) (1 + r^*) &\geq 0 \end{aligned}$$

The solution is:

$$D_s = \begin{cases} \frac{2\psi_s + \frac{\psi_s^I + \psi_s}{\psi_s^I \beta p_x} - \sqrt{4\psi_s^2 + \left( \frac{\psi_s^I + \psi_s}{\psi_s^I \beta p_x} \right)^2}}{\frac{2\psi_s(\psi_s^I + \psi_s)}{\psi_s^I \beta p_x}} & \text{if } p_x \geq \bar{p}_x^s \\ 0 & \text{if } 0 < p_x < \bar{p}_x^s, \end{cases}$$

where $\bar{p}_x^s$ is a price cutoff. The FOCs imply that $D_s^I = \frac{\psi_s}{\psi_s^I} D_s$. Substituting the optimal debt into the first (binding) constraint characterizes labor and production, where $X_s = \ln L$ denotes production by the seller. Substituting optimal debt and labor into the objective function yields the maximized value of a seller with cost draw $(\psi_s, \psi_s^I)$, which we denote by $\Gamma_s^I(\psi_s, \psi_s^I)$. This value is net of the fixed cost of market access, $f^I$.

### A.1.2 Buyer's General Problem

Let $I_b^I$ be an indicator function that takes the value of 1 whenever the buyer has access to the foreign-currency debt market, and zero otherwise. The buyer's general problem is:

$$\max_{D_b, I_b^I \cdot D_b^I, X_b \geq 0} D_b + I_b^I \cdot D_b^I - p_x X_b - \psi_b D_b^2 - \psi_b^I (I_b^I \cdot D_b^I)^2 + \beta E_z \left[ z X_b - \left( D_b + I_b^I \cdot D_b^I \right) (1 + r^*) \right] - I_b^I \cdot f^I$$

subject to:

$$
\begin{aligned}
D_b + I_b^I \cdot D_b^I - p_x X_b - \psi_b D_b^2 - \psi_b^I (I_b^I \cdot D_b^I)^2 &\geq 0 \\
\underline{z} X_b - (D_b + I_b^I \cdot D_b^I)(1 + r^*) &\geq 0
\end{aligned}
$$

The solution for $D_b$ is identical to the one in the main text because of the linearity in the buyer's problem. When $I_b^I = 1$, the optimal amount of foreign currency debt is given by $D_b^I = \frac{\psi_b}{\psi_b^I} D_b$. The optimal quantity of intermediate good purchased, $X_b$, follows from the constraints. The maximized value is denoted by $\Gamma_b^I(\psi_b, \psi_b^I)$ when $I_b^I = 1$, and by $\Gamma_b(\psi_b, \psi_b^I)$ otherwise.

## A.2 Bilateral Matches

We maintain the assumption that sellers make take-it-or-leave-it offers to buyers. Let $I_s^I$ be an indicator function that takes the value of 1 whenever the seller has access to the foreign-currency debt market, and zero otherwise. The seller solves the following problem:

$$
\begin{aligned}
\max_{D_s \geq 0, D_b \geq 0, I_s^I \cdot D_s^I \geq 0, I_b^I \cdot D_b^I \geq 0, L \geq 1, \underline{T}, \bar{T}, A_b, B_s, B_b} \; & D_s + I_s^I \cdot D_s^I + A_b - wL - \xi \mathbf{1}\left\{\frac{\bar{T}}{\underline{T}} \neq 1\right\} \\
& - \psi_s (D_s)^2 - \psi_s^I \left(I_s^I \cdot D_s^I\right)^2 \\
& + \beta \left[\left(B_s - D_s - I_s^I \cdot D_s^I\right)(1 + r^*) + \tilde{T}\right] \\
& - \Gamma_s\left(\psi_s, \psi_s^I\right) - I_s^I [\Gamma_s^I \left(\psi_s, \psi_s^I\right) - \Gamma_s\left(\psi_s, \psi_s^I\right) + f^I]
\end{aligned}
$$

subject to:

$$\lambda_1: \; D_b + I_b^I \cdot D_b^I - \psi_b \left(D_b\right)^2 - \psi_b^I \left(I_b^I \cdot D_b^I\right)^2 - A_b +$$

$$\beta \left[\left(B_b - D_b - I_b^I \cdot D_b^I\right)\left(1 + r^*\right) - \tilde{T} + \tilde{z} \ln L\right] - \Gamma \geq 0 \tag{31}$$

$$\lambda_2: \; B_b = D_b + I_b^I \cdot D_b^I - \psi_b \left(D_b\right)^2 - \psi_b^I \left(I_b^I \cdot D_b^I\right)^2 - A_b \tag{32}$$

$$\lambda_3: \; B_s = D_s + I_s^I \cdot D_s^I - \psi_s \left(D_s\right)^2 - \psi_s^I \left(I_s^I \cdot D_s^I\right)^2 + A_b - wL - \xi \mathbf{1}\left\{\frac{\bar{T}}{\underline{T}} \neq 1\right\} \tag{33}$$

$$\lambda_4: \; B_s \geq 0 \tag{34}$$

$$\lambda_5: \; B_b \geq 0 \tag{35}$$

$$\lambda_6: \; \left(B_s - D_s - I_s^I \cdot D_s^I\right)\left(1 + r^*\right) + \underline{T} \geq 0 \tag{36}$$

$$\lambda_7: \; \left(B_b - D_b - I_b^I \cdot D_b^I\right)\left(1 + r^*\right) - \underline{T} + \underline{z} \ln L \geq 0 \tag{37}$$

$$\lambda_8: \; \left(B_b - D_b - I_b^I \cdot D_b^I\right)\left(1 + r^*\right) - \bar{T} + \bar{z} \ln L \geq 0, \tag{38}$$

where $\Gamma \equiv \Gamma_b\left(\psi_b, \psi_b^I\right) + I_b^I[\Gamma_b^I\left(\psi_b, \psi_b^I\right) - \Gamma_b\left(\psi_b, \psi_b^I\right) + f^I]$ and the dependence on the buyer's cost draw is suppressed for ease of notation.

Let $\lambda_i$ denote the multiplier associated with constraints $i = 1, ..., 8$ given by expressions (31)-(38).

#### A.2.1 Case 1: Unconstrained agents

We assume that the debt repayment constraints (36)-(38) do not bind in any state of the world. As in the baseline case, this implies that $\bar{T}_1 = \underline{T_1}$ is feasible. Since $\xi > 0$, it is optimal to set $T_1 = \bar{T}_1 = \underline{T_1}$. That is, since insurance is costly to provide, in the equilibrium where agents are not credit constrained and therefore not in need of insurance, trade credit is equalized across states of the world. In addition, the seller always has the incentive to extract all the surplus from the buyer, which implies that constraint (31) is binding. In order to understand how debt levels and production behave in this equilibrium, denote by $\lambda_{i,1}$ the multiplier for the constraint in expression $(i)$ above, take FOCs, and simplify to obtain:

$$L_1 = \frac{\tilde{z}\beta}{(1-\tilde{\lambda}_1)w} \tag{39}$$

$$D_{s,1} = \frac{\tilde{\lambda}_1}{2\psi_s\left(\tilde{\lambda}_1 - 1\right)} \tag{40}$$

$$D_{b,1} = \frac{\tilde{\lambda}_1}{2\psi_b\left(\tilde{\lambda}_1 - 1\right)}, \tag{41}$$

where $\tilde{\lambda}_1 \equiv \lambda_{3,1}$. Expressions (40) and (41) imply that, in order to maximize production, each agent will borrow according to her borrowing capacity relative to her partner: $D_{s,1}/D_{b,1} = \psi_b/\psi_s$.

When $I_s^I = I_b^I = 1$, the FOCs for foreign-currency debt yield:

$$D_{s,1}^I = \frac{\tilde{\lambda}_1}{2\psi_s^I\left(\tilde{\lambda}_1 - 1\right)}$$

$$D_{b,1}^I = \frac{\tilde{\lambda}_1}{2\psi_b^I\left(\tilde{\lambda}_1 - 1\right)}$$

Hence, foreign-currency debt is proportional to domestic-currency debt for each agent, $D_{s,1}/D_{s,1}^I = \psi_s^I/\psi_s$ and $D_{b,1}/D_{b,1}^I = \psi_{b,1}^I/\psi_b$.

Using these equalities, together with expression (39) into constraints (32) and (33) yields a unique solution for the multiplier $\tilde{\lambda}_1 < 1$ given by:

$$\tilde{\lambda}_1 = 1 - \frac{2}{\Psi}\left[\tilde{z}\beta + \sqrt{\left(\frac{\Psi}{2}\right)^2 + (\tilde{z}\beta)^2}\right] \tag{42}$$

The parameter $\Psi$ can take on four different values, which depend on the source of financing for each agent:

$$\Psi = \begin{cases} \frac{1}{\psi_s} + \frac{1}{\psi_b} & \text{if } I_s^I = I_b^I = 0 \\ \frac{1}{\psi_s} + \frac{1}{\psi_b} + \frac{1}{\psi_s^I} + \frac{1}{\psi_b^I} & \text{if } I_s^I = I_b^I = 1 \\ \frac{1}{\psi_s} + \frac{1}{\psi_b} + \frac{1}{\psi_s^I} & \text{if } I_s^I = 1 \,\&\, I_b^I = 0 \\ \frac{1}{\psi_s} + \frac{1}{\psi_b} + \frac{1}{\psi_b^I} & \text{if } I_s^I = 0 \,\&\, I_b^I = 1 \end{cases}$$

In the above expression, the first line corresponds to a situation where both the buyer and the seller borrow in domestic currency only. This is precisely the solution given in the main text. The second line describes the solution when buyers and sellers borrow both in domestic and in foreign currency. The third (fourth) line characterizes the solution when the buyer (seller) borrows in domestic currency only, while their partner borrows in both domestic and foreign currency.

Having found the optimal $\tilde{\lambda}_1$, we can substitute expression (42) into expressions (39), (40), (41) and constraint (31) which holds with equality, in order to obtain the optimal levels of $L_1, D_{s,1}, D_{b,1}$ and $T_1$, respectively. The foreign-currency debt allocations are simply proportional to the domestic-currency ones.

#### A.2.2 Case 2: Constrained agents and no insurance

When debt repayment constraints begin to bind, the optimal level of trade credit $T_1$ above is no longer feasible. Since insurance is costly ($\xi > 0$), debt constrained sellers may still find it optimal to not provide insurance to buyers. In this case, $T_2 = \bar{T}_2 = \underline{T_2}$, where $T_2$ is the transfer under the constrained scenario. Further, as discussed above, constraints (37) and (38) cannot be jointly binding. As in the baseline model, we consider the case where constraint (37) is the binding one. Similar arguments show that constraints (36) and (31) cannot be binding.

If $\lambda_{i,2}$ is the multiplier for the constraint in expression ($i$) above, the above discussion implies that $\lambda_{8,2} = \lambda_{6,2} = \lambda_{1,2} = 0$. Taking FOCs and simplifying yields:

$$L_2 = \frac{\underline{z}\beta}{(1 - \tilde{\lambda}_2)w} \tag{43}$$

$$D_{s,2} = \frac{\tilde{\lambda}_2}{2\psi_s\left(\tilde{\lambda}_2 - 1\right)} \tag{44}$$

$$D_{b,2} = \frac{\tilde{\lambda}_2}{2\psi_b\left(\tilde{\lambda}_2 - 1\right)}, \tag{45}$$

where $\tilde{\lambda}_2 \equiv \lambda_{3,2}$.

Even when agents are constrained, in order to maximize production, each agent will borrow according to her borrowing capacity relative to her partner, which can be seen from expressions (44) and (45): $D_{s,2}/D_{b,2} = \psi_b/\psi_s$. Similarly, the FOCs for foreign debt imply the proportionality result: $D_{s,1}/D^I_{s,1} = \psi^I_s/\psi_s$ and $D_{b,1}/D^I_{b,1} = \psi^I_{b,1}/\psi_b$. Using these equalities, together with expression (43) into constraints (32) and (33) allows to obtain a unique solution for the multiplier $\tilde{\lambda}_2 < 1$ given by:

$$\tilde{\lambda}_2 = 1 - \frac{2}{\Psi}\left[\underline{z}\beta + \sqrt{\left(\frac{\Psi}{2}\right)^2 + (\underline{z}\beta)^2}\right] \tag{46}$$

Having found the optimal $\tilde{\lambda}_2$, we can substitute expression (46) into expressions (43), (44), (45) and constraint (37) which holds with equality (imposing that $T_2 = \bar{T}_2 = \underline{T_2}$), in order to obtain the optimal levels of $L_2, D_{s,2}, D_{b,2}$ and $T_2$, respectively. The foreign-currency debt levels follow from the proportionality result.

Debt levels and production levels between Cases 1 and 2 are ordered as before.

### A.2.3 Case 3: Constrained agents and insurance; buyer is constrained in bad state

The next two cases are more interesting from the point of view of this paper because they describe situations in which transfers are state contingent, and therefore, trade credit provides agents with insurance. Denote by $\lambda_{i,3}$ the multiplier for the constraint in expression $(i)$ above and combine the FOCs for $\bar{T}_3$ and $\underline{T_3}$ to obtain:

$$\lambda_{8,3}\frac{p}{1-p} = \lambda_{7,3} - \lambda_{6,3}$$

As discussed above, constraints (37) and (38) cannot be jointly binding. We consider the case where constraint (37) is the binding one. Then, $\lambda_{8,3} = 0$ implies that $\lambda_{6,3} = \lambda_{7,3}$.

Following the same steps as in the baseline model yields an implicit equation for $D_{s,3}$:

$$(1-\psi_s D_{s,3})\,\psi_s D_{s,3}\Psi = we^{\frac{\psi_s D_{s,3}}{\beta \underline{z}}\Psi} + \xi \tag{47}$$
$$\text{if} \quad \frac{\Psi}{4} \geq we^{\frac{\Psi}{2\beta \underline{z}}} + \xi$$

Having found that, $D_{b,3}, D^I_{b,3}, D^I_{s,3}$ follow trivially from the proportionality result, and $\bar{T}_3, \underline{T_3}$ and $L_3$ follow from expressions (48), (49) and (50), respectively:

$$\frac{\Gamma}{\beta} = \tilde{z}\ln L_3 - p\underline{T_3} - (1-p)\bar{T}_3 - (D_{b,3} + I^I_b \cdot D^I_{b,3})(1+r^*) \tag{48}$$

$$\beta\underline{T_3} = D_{s,3} + I^I_s \cdot D^I_{s,3} \tag{49}$$

$$D_{b,3} + D_{s,3} + I^I_b \cdot D^I_{b,3} + I^I_s \cdot D^I_{s,3} = \frac{\underline{z}\ln L_3}{1+r^*} \tag{50}$$

### A.2.4 Case 4: Constrained agents and insurance; buyer is constrained in good state

As discussed above, constraints (37) and (38) cannot be jointly binding. Now, we consider the case where constraint (38) is the binding constraint for the buyer, while constraint (36) continues to bind for the seller. Constraints (36) and (38) yield the values for $\underline{T_4}$ and $\bar{T}_4$, respectively, which together yield:

$$\tilde{T_4} = \frac{\psi_s D_{s,4}}{\beta}\left(p\Psi - \frac{1}{\psi_b} - I_b^I \cdot \frac{1}{\psi_b^I}\right) + (1-p)\,\bar{z}\ln L_4 \tag{51}$$

Following the same steps as in the baseline model yields the wage bill:

$$wL_4 = we^{\frac{\psi_s D_{s,4}}{\beta \underline{z}}\Psi + \frac{\Gamma}{p\underline{z}\beta}} \tag{52}$$

The optimal level of debt $D_{s,4}$ solves the following implicit function:

$$\begin{aligned} &(1-\psi_s D_{s,3})\,\psi_s D_{s,3}\Psi = we^{\frac{\psi_s D_{s,3}}{\beta \underline{z}}\Psi + \frac{\Gamma}{\beta p \underline{z}}} + \xi \\ \text{if}\quad &\frac{\Psi}{4} \geq we^{\frac{\Psi}{2\beta\underline{z}} + \frac{\Gamma}{\beta p \underline{z}}} + \xi \end{aligned} \tag{53}$$

Debt levels satisfy the proportionality condition as before and $\underline{T_4}, \bar{T}_4$ and $L_4$ follow from constraints (36) and (38) and expression (52), respectively.

The solutions to cases 3 and 4 closely resemble each other. They both yield state contingent transfers. The level of the transfers, as well as the level of debt and production varies across the cases. Similar arguments as in the main text can be used to show that case 4 yields lower levels of debt and production than case 3.

# B Proofs and Derivations

## B.1 Dealings with Intermediary Algebra

### B.1.1 Seller

The FOC's of the seller's problem yield:

$$D_s = \frac{2\psi_s + \frac{1+r^*}{p_x} \pm \sqrt{4\psi_s^2 + \left(\frac{1+r^*}{p_x}\right)^2}}{2\psi_s \frac{1+r^*}{p_x}}$$

The larger of the two roots violates the restriction imposed by the log production function. To see this, substitute $L$ into the production function, $\ln L$, to obtain the following expression: $\ln(D_s - \psi_s D_s^2) - \ln(w)$. Production is well defined when $D_s - \psi_s D_s^2 \geq w$. The larger of the two roots violates this inequality. The smaller of the two roots satisfies the inequality if the equilibrium price, $p_x$, is high enough:

$$p_x \geq (1 + r^*)\sqrt{\frac{2\psi_s}{w(2\psi_2 w - 1)}}$$

Note that when the second constraint is binding, the seller's value function is exactly zero. Hence, in this case, optimal debt and production are also zero. This solution is summarized in the main body of the paper.

### B.1.2 Buyer

Since the first constraint must hold with equality, we can substitute it into the objective function to obtain:

$$\max_{D_b \geq 0} \beta E_z \left[\frac{z}{p_x}\left(D_b - \psi_b D_b^2\right) - D_b\left(1 + r^*\right)\right]$$

subject to:

$$\frac{\underline{z}}{p_x}\left(D_b - \psi_b D_b^2\right) - D_b\left(1+r^*\right) \quad \geq \quad 0$$

The FOCs yield the following optimal solution, as a function of the Lagrange multiplier $\lambda$:

$$D_b = \frac{1}{2\psi_b}\left[1 - \frac{p_x\left(1+\lambda\left(1+r^*\right)\right)}{\beta\tilde{z}+\lambda\underline{z}}\right]$$

There are two cases; one where the constraint does not bind and one where it binds. In the first case, the Lagrange multiplier must be zero, so

$$D_b = \frac{1}{2\psi_b}\left[1 - \frac{p_x\left(1+r^*\right)}{\tilde{z}}\right] \tag{54}$$

which is non-negative whenever the equilibrium price is low enough $p_x \leq \frac{\tilde{z}}{1+r^*}$. In the second case, we obtain $D_b$ directly from the second constraint, which holds with equality. The solution is given by:

$$D_b \quad = \quad \frac{1}{\psi_b}\left[1 - \frac{p_x\left(1+r^*\right)}{\underline{z}}\right] \tag{55}$$

which is non-negative whenever the equilibrium price is even lower, $p_x \leq \frac{\underline{z}}{1+r^*}$.

Which of the two expressions characterizes the optimal amount of debt depends on feasibility. The objective function of the buyer is a quadratic equation which has an inverted U shape. The limits on the positive and negative spectrum are both negative infinity. It attains a maximum when $D_b$ is given by the unconstrained solution (54), which is non-negative under the aforementioned parameter restrictions. The solution to the constrained problem in (55) may be higher or lower than the solution to the unconstrained problem. However, when it is higher, the constrained solution is sub-optimal, since the max is achieved at the unconstrained solution. Thus, the solution to the buyer's problem is given by the unconstrained solution, unless it is infeasible, in which case it is given by the constrained solution. Substituting the unconstrained solution (54) into the constraint shows that the unconstrained solution is infeasible when $p_x > \frac{\underline{z}}{(2-\frac{\underline{z}}{\tilde{z}})(1+r^*)}$. Hence, the solution to the buyer's problem is given by expression (55) whenever $\frac{\underline{z}}{1+r^*} \geq p_x > \frac{\underline{z}}{(2-\frac{\underline{z}}{\tilde{z}})(1+r^*)}$ and by expression (54) whenever $p_x \leq \frac{\underline{z}}{(2-\frac{\underline{z}}{\tilde{z}})(1+r^*)}$. This solution is summarized in the main body of the paper.

Next we show that $\Gamma_b(\psi_b)$ is decreasing in $\psi_b$. In the case where the buyer is unconstrained, their maximized value is:

$$\Gamma_b(\psi_b) = \frac{1}{4\psi_b}\left[1 - \frac{p_x}{\beta\tilde{z}}\right]\left[\frac{\tilde{z}}{p_x} - \frac{1}{\beta}\right] \tag{56}$$

In the case where the buyer is constrained, their maximized value is

$$\Gamma_b(\psi_b) = \frac{1}{\psi_b}\left[1 - \frac{p_x}{\beta\underline{z}}\right]\left[\frac{\tilde{z}}{\underline{z}} - 1\right] \tag{57}$$

These values are positive as long as $\beta\tilde{z} > p_x$ ($\beta\underline{z} > p_x$ when the buyer is constrained) which ensures that debt levels are non-negative. Taking the derivative of each expression with respect to $\psi_b$ yields the result.

### B.2 Case 1 Algebra

The FOCs to the seller's problem are given by:

$$
\begin{aligned}
[\bar{T}_1]: &\quad (1-\lambda_1)\,\beta\,(1-p) = 0 \quad \Rightarrow \quad \lambda_{1,1} = 1 \\
[\underline{T_1}]: &\quad (1-\lambda_{1,1})\,\beta p = 0 \quad \Rightarrow \quad \lambda_{1,1} = 1 \\
[L_1]: &\quad (\lambda_{3,1}-1)w + \beta\lambda_{1,1}\frac{\tilde{z}}{L_1} = 0 \quad \Rightarrow L_1 = \frac{\tilde{z}\beta}{(1-\lambda_{3,1})w} \\
[A_{b,1}]: &\quad 1-\lambda_{1,1}+\lambda_{2,1}-\lambda_{3,1} = 0 \quad \Rightarrow \lambda_{2,1} = \lambda_{3,1} \\
[D_{s,1}]: &\quad -2\psi_s D_{s,1} - \lambda_{3,1}\,(1-2\psi_s D_{s,1}) = 0 \quad \Rightarrow D_{s,1} = \frac{\lambda_{3,1}}{2\psi_s\,(\lambda_{3,1}-1)} \\
[D_{b,1}]: &\quad 2\psi_s D_{b,1}\,(\lambda_{2,1}-\lambda_{1,1}) - \lambda_{2,1} = 0 \quad \Rightarrow D_{b,1} = \frac{\lambda_{2,1}}{2\psi_b\,(\lambda_{2,1}-\lambda_{1,1})} = \frac{\lambda_{3,1}}{2\psi_b\,(\lambda_{3,1}-1)} \\
[B_{s,1}]: &\quad 1+\lambda_{3,1}+\lambda_{4,1} = 0 \\
[B_{b,1}]: &\quad 1+\lambda_{2,1}+\lambda_{5,1} = 0 \quad \Rightarrow \lambda_{4,1} = \lambda_{5,1}
\end{aligned}
$$

Let $\tilde{\lambda}_1 = \lambda_{3,1}$. Sum constraints (2) and (3) to get:

$$-D_{b,1} + \psi_b\,(D_{b,1})^2 - D_{s,1} + wL_1 + \psi_s\,(D_{s,1})^2 \;=\; 0$$

Substitute out the debt levels and labor to obtain:

$$\frac{1}{4}\left(\frac{\psi_b+\psi_s}{\psi_b\psi_s}\right)\tilde{\lambda}_1^2 - \left(\frac{1}{2}\left(\frac{\psi_s+\psi_b}{\psi_b\psi_s}\right) - \tilde{z}\beta\right)\tilde{\lambda}_1 - \tilde{z}\beta \;=\; 0$$

This is a quadratic equation in $\tilde{\lambda}_1$ whose only root that satisfies $\tilde{\lambda}_1 < 1$ is given in the main text of the paper.

In order for this case to be feasible, the solution needs to satisfy constraints (6)-(8). If constraint (7) is satisfied, then (8) holds trivially. If constraint (6) is satisfied, then the maximized value for the seller is non-negative, which is another necessary condition.

The optimal unconstrained allocation satisfies constraint (6) only if

$$\tilde{z}\ln\left(\frac{\tilde{z}\beta}{w(1-\tilde{\lambda}_1)}\right)-\frac{\psi_s+\psi_b}{2\beta\psi_b\psi_s}\frac{\tilde{\lambda}_1}{\tilde{\lambda}_1-1}\geq\Gamma_b(\psi_b)/\beta \tag{58}$$

The optimal unconstrained allocation satisfies constraint (7) only if

$$\Gamma_b(\psi_b)/\beta\geq(\tilde{z}-\underline{z})\ln\left(\frac{\tilde{z}\beta}{w(1-\tilde{\lambda}_1)}\right) \tag{59}$$

The two restrictions are jointly satisfied only if

$$\underline{z}\ln\left(\frac{\tilde{z}\beta}{w(1-\tilde{\lambda}_1)}\right)\geq\frac{\psi_s+\psi_b}{2\beta\psi_b\psi_s}\frac{\tilde{\lambda}_1}{\tilde{\lambda}_1-1} \tag{60}$$

Substituting the solution for $\tilde{\lambda}_1$ into (60) yields the parameter restriction in the main text.

Finally, due to the functional form for the production function, it must be that $L_1\geq 1$, which requires that $\tilde{z}\beta\geq w(1-\tilde{\lambda}_1)$. Restriction (60), however, is more strict than this restriction because $\tilde{\lambda}_1/(\tilde{\lambda}_1-1)\geq 0$, so it is trivially satisfied.

## B.3 Case 2 Algebra

The FOCs to the seller's problem are given by:

$$[L_2]: \qquad (\lambda_{3,2}-1)w + \beta\lambda_{1,2}\frac{\tilde{z}}{L_2} + \frac{\underline{z}}{L_2}\lambda_{7,2} + \frac{\underline{z}}{L_2}\lambda_{8,2} = 0 \quad \Rightarrow L_2 = \frac{\beta\lambda_{1,2}\tilde{z} + \underline{z}\lambda_{7,2} + \lambda_{8,2}\bar{z}}{(1-\lambda_{3,2})w}$$

$$[B_{s,2}]: \qquad 1 + \lambda_{3,2} + \lambda_{4,2} + \frac{\lambda_{6,2}}{\beta} = 0$$

$$[B_{b,2}]: \qquad \lambda_{1,2} + \lambda_{2,2} + \lambda_{5,2} + \frac{\lambda_{7,2}}{\beta} = 0$$

$$[A_{b,2}]: \qquad 1 - \lambda_{1,2} + \lambda_{2,2} - \lambda_{3,2} = 0$$

$$[D_{s,2}]: \qquad -2\psi_s D_{s,2} - \lambda_{3,2}\left(1 - 2\psi_s D_{s,2}\right) - \frac{\lambda_{6,2}}{\beta} = 0 \quad \Rightarrow D_{s,2} = \frac{\lambda_{3,2} + \frac{\lambda_{6,2}}{\beta}}{2\psi_s\left(\lambda_{3,2}-1\right)} = \frac{1+\lambda_{4,2}}{2\psi_s\left(1-\lambda_{3,2}\right)}$$

$$[D_{b,2}]: \qquad 2\psi_b D_{b,2}\left(\lambda_{2,2} - \lambda_{1,2}\right) - \lambda_{2,2} - \frac{\lambda_{7,2}}{\beta} = 0 \quad \Rightarrow D_{b,2} = \frac{\lambda_{2,2} + \frac{\lambda_{7,2}}{\beta}}{2\psi_b\left(\lambda_{2,2}-\lambda_{1,2}\right)} = \frac{\lambda_{1,2}+\lambda_{5,2}}{2\psi_b\left(1-\lambda_{3,2}\right)}$$

$$[T_2]: \qquad \beta\left(1-\lambda_{1,2}\right) + \lambda_{6,2} - \lambda_{7,2} = 0$$

Let $\lambda_{8,2} = \lambda_{6,2} = \lambda_{1,2} = 0$. Then we need to solve the following system:

$$\begin{aligned}
L_2 &= \frac{\underline{z}\lambda_{7,2}}{(1-\lambda_{3,2})w} \\
0 &= 1 + \lambda_{3,2} + \lambda_{4,2} \\
0 &= \lambda_{2,2} + \lambda_{5,2} + \frac{\lambda_{7,2}}{\beta} \\
0 &= 1 + \lambda_{2,2} - \lambda_{3,2} \\
D_{s,2} &= \frac{\lambda_{3,2}}{2\psi_s\left(\lambda_{3,2}-1\right)} \\
D_{b,2} &= \frac{\lambda_{5,2}}{2\psi_b\left(1-\lambda_{3,2}\right)} \\
\lambda_{7,2} &= \beta \\
0 &= -D_{b,2} + A_{b,2} + \psi_b\left(D_{b,2}\right)^2 \\
0 &= -D_{s,2} - A_{b,2} + wL_2 + \psi_s\left(D_{s,2}\right)^2 \\
0 &= -D_{b,2}(1+r^*) - T_2 + \underline{z}\ln L_2
\end{aligned}$$

The system simplifies to:

$$\begin{aligned}
L_2 &= \frac{\underline{z}\beta}{(1-\lambda_{3,2})w} \\
0 &= 1 + \lambda_{3,2} + \lambda_{4,2} \quad \Rightarrow \quad \lambda_{4,2} = -\left(1+\lambda_{3,2}\right) \\
0 &= 1 + \lambda_{2,2} + \lambda_{5,2} \quad \Rightarrow \quad \lambda_{5,2} = -\left(1+\lambda_{2,2}\right) = -\lambda_{3,2} \\
0 &= 1 + \lambda_{2,2} - \lambda_{3,2} \quad \Rightarrow \quad \lambda_{2,2} = \left(\lambda_{3,2}-1\right) \\
D_{s,2} &= \frac{\lambda_{3,2}}{2\psi_s\left(\lambda_{3,2}-1\right)} \\
D_{b,2} &= \frac{\lambda_{5,2}}{2\psi_b\left(1-\lambda_{3,2}\right)} \\
0 &= -D_{b,2} + A_{b,2} + \psi_b\left(D_{b,2}\right)^2 \\
0 &= -D_{s,2} - A_{b,2} + wL_2 + \psi_s\left(D_{s,2}\right)^2 \\
0 &= -D_{b,2}(1+r^*) - T_2 + \underline{z}\ln L_2
\end{aligned}$$

Let $\tilde{\lambda}_2 = \lambda_{3,2}$. To solve for this object, substitute out $A_{b,2}$ in the second to the last equation:

$$0 = -D_{b,2} - D_{s,2} + \psi_b (D_{b,2})^2 + \psi_s (D_{s,2})^2 + wL_2$$

Substitute out the debt levels and labor to obtain:

$$0 = \frac{1}{4}\left(\frac{\psi_s + \psi_b}{\psi_s \psi_b}\right)\tilde{\lambda}^2 + \left[\underline{z}\beta - \frac{1}{2}\left(\frac{\psi_s + \psi_b}{\psi_s \psi_b}\right)\right]\tilde{\lambda}_2 - \underline{z}\beta$$

This is a quadratic equation in $\tilde{\lambda}_2$ whose only root that satisfies $\tilde{\lambda}_2 < 1$ is given in the main text of the paper.

By construction, the solution satisfies all the constraints, and the seller's maximized value is non-negative due to constraint (6). An additional restriction is that production is non-negative, which requires that $\underline{z}\beta \geq (1 - \lambda_2)w$. Substituting out $\tilde{\lambda}_2$ in this inequality yields the parameter restriction in the main text.

### B.3.1 Ruling out Case 5

Consider the problem in Case 2 and the FOCs in the above section, before imposing values for the multipliers. Just as in Case 2, constraint (8) can never bind given that constraint (7) with a flat T is more restrictive than constraint (8). In Case 2, we assumed that constraints (1), (6) and (8) did not bind, so only (7) was binding. Suppose that we assume that constraint (6) binds instead and that $\lambda_{1,2} = \lambda_{8,2} = \lambda_{7,2} = 0$. Replacing these restrictions in the first FOC we get:

$$[L]: \qquad L_2 = \frac{0}{(1 - \lambda_{3,2})w} = 0$$

In fact, if constraint (6) binds, the value of the seller is 0 because T is flat. Therefore, this case cannot exist. Even if constraint (1) is not binding, the seller would not be willing to participate in this match.

## B.4 Case 3 algebra

Taking FOCs of the seller's problem yields:

$$
\begin{aligned}
[L_3]: &\qquad (\lambda_{3,3}-1)w+\beta\lambda_{1,3}\frac{\tilde{z}}{L_3}+\frac{\underline{z}}{L_3}\lambda_{7,3}+\frac{\bar{z}}{L}\lambda_{8,3}=0 \quad \Rightarrow L_3=\frac{\beta\lambda_{1,3}\tilde{z}+\underline{z}\lambda_{7,3}+\bar{z}\lambda_{8,3}}{(1-\lambda_{3,3})w}\\
[B_{s,3}]: &\qquad 1+\lambda_{3,3}+\lambda_{4,3}+\frac{\lambda_{6,3}}{\beta}=0\\
[B_{b,3}]: &\qquad \lambda_{1,3}+\lambda_{2,3}+\lambda_{5,3}+\frac{\lambda_{7,3}+\lambda_{8,3}}{\beta}=0\\
[A_{b,3}]: &\qquad 1-\lambda_{1,3}+\lambda_{2,3}-\lambda_{3,3}=0\\
[D_{s,3}]: &\qquad -2\psi_s D_{s,3}-\lambda_{3,3}\left(1-2\psi_s D_{s,3}\right)-\frac{\lambda_{6,3}}{\beta}=0 \quad \Rightarrow D_{s,3}=\frac{\lambda_{3,3}+\frac{\lambda_{6,3}}{\beta}}{2\psi_s\left(\lambda_{3,3}-1\right)}=\frac{1+\lambda_{4,3}}{2\psi_s\left(1-\lambda_{3,3}\right)}\\
[D_{b,3}]: &\qquad 2\psi_b D_{b,3}\left(\lambda_{2,3}-\lambda_{1,3}\right)-\lambda_{2,3}-\frac{\lambda_{7,3}+\lambda_{8,3}}{\beta}=0 \quad \Rightarrow D_{b,3}=\frac{\lambda_{2,3}+\frac{\lambda_{7,3}+\lambda_{8,3}}{\beta}}{2\psi_b\left(\lambda_{2,3}-\lambda_{1,3}\right)}=\frac{\lambda_{1,3}+\lambda_{5,3}}{2\psi_b\left(1-\lambda_{3,3}\right)}\\
\left[\bar{T}_3\right]: &\qquad \beta\left(1-p\right)\left(1-\lambda_{1,3}\right)-\lambda_{8,3}=0\\
\left[\underline{T}_3\right]: &\qquad \beta p\left(1-\lambda_{1,3}\right)+\lambda_{6,3}-\lambda_{7,3}=0
\end{aligned}
$$

Combining FOCs with the constraints yields a system of 13 equations and 13 unknowns:

$$
\begin{aligned}
L_3 &= \frac{\beta\lambda_{1,3}\tilde{z} + \underline{z}\lambda_{7,3} + \bar{z}\lambda_{8,3}}{(1-\lambda_{3,3})w} \\
0 &= 1 + \lambda_{3,3} + \lambda_{4,3} + \frac{\lambda_{6,3}}{\beta} \quad \text{then} \quad (4) \quad \Rightarrow \quad \lambda_{4,3} = \lambda_{5,3} \\
0 &= \lambda_{1,3} + \lambda_{2,3} + \lambda_{5,3} + \frac{\lambda_{7,3}}{\beta} \quad \text{then} \quad (4) \quad \Rightarrow \quad \lambda_{4,3} = \lambda_{5,3} \\
0 &= 1 - \lambda_{1,3} + \lambda_{2,3} - \lambda_{3,3} \quad \text{then} \quad (3) \quad \Rightarrow \quad \lambda_{2,3} = \lambda_{3,3} \\
D_{s,3} &= \frac{1+\lambda_{4,3}}{2\psi_s\,(1-\lambda_{3,3})} \\
D_{b,3} &= \frac{\lambda_{1,3}+\lambda_{5,3}}{2\psi_b\,(1-\lambda_{3,3})} \\
0 &= \beta\,(1-p)\,(1-\lambda_{1,3}) \quad \text{then} \quad (1) \quad \Rightarrow \quad \lambda_{1,3} = 1 \\
0 &= \beta p\,(1-\lambda_{1,3}) + \lambda_{6,3} - \lambda_{7,3} \quad \text{then} \quad (2) \quad \Rightarrow \quad \lambda_{7,3} = \lambda_{6,3} \\
\frac{\Gamma_b}{\beta} &= \tilde{z}\ln L_3 - \tilde{T}_3 - D_{b,3}(1+r^*) \\
A_{b,3} &= D_{b,3} - \psi_b\,(D_{b,3})^2 \\
0 &= -\,(D_{s,3}+D_{b,3}) + wL_3 + \psi_s\,(D_{s,3})^2 + \psi_b\,(D_{b,3})^2 + \xi \\
\underline{T_3} &= D_{s,3}(1+r^*) \\
D_{b,3} + D_{s,3} &= \frac{\underline{z}\ln L_3}{1+r^*}
\end{aligned}
$$

The system reduces to the 8 equations and 8 unknowns in the main text. The solution method involves first characterizing $D_{s,3}$ via the implicit function in expression (53). The quadratic equation in expression (53) has either no solution or two solutions (with a knife edge case of a unique solution). The max of the LHS is $D_{s,3} = \frac{1}{2\psi_s}$. Evaluating the LHS and the RHS at this value obtains:

$$
\begin{aligned}
LHS &= \frac{1}{4}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right) \\
RHS &= we^{\frac{\frac{1+r^*}{2}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)}{\underline{z}}}+\xi
\end{aligned}
$$

To ensure the existence of a pair of solutions, the following parameter restriction is necessary:

$$
\frac{1}{4}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right) \geq we^{\frac{1+r^*}{2\underline{z}}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)}+\xi \tag{61}
$$

Given the two roots, the optimal level of $D_{s,3}$ is the one that is associated with a higher value function for the seller. To see when that occurs, substitute expressions (18), (20) and (21) into the seller's objective function to obtain

$$
V_{s,3} = \ln L_3(1-p)(\bar{z}-\underline{z}) - \Gamma(\psi_b)/\beta
$$

Clearly $V_{s,3}$ is maximized when $L_3$ is maximized. Since, according to expression (21), labor is increasing in the amount borrowed, it must be that the higher value of debt is the optimal solution to this case. Substituting expression (21) into $V_{s,3}$ yields

$$
V_{s,3} = D_{b,3}(1+r^*)\frac{\psi_s+\psi_b}{\psi_s}(1-p)\left(\frac{\bar{z}}{\underline{z}}-1\right) - \Gamma(\psi_b)/\beta
$$

We will compare this value to the value of the case 4 below.

## B.5 Case 4 Algebra

Taking FOCs of the seller's problem yields:

$$
\begin{aligned}
[L_4]: &\qquad L_4 = \frac{\beta\lambda_{1,4}\tilde{z} + \bar{z}\lambda_{8,4}}{(1-\lambda_{3,4})w} \\
[B_{s,4}]: &\qquad 1 + \lambda_{3,4} + \lambda_{4,4} + \frac{\lambda_{6,4}}{\beta} = 0 \\
[B_{b,4}]: &\qquad \lambda_{1,4} + \lambda_{2,4} + \lambda_{5,4} + \frac{\lambda_{8,4}}{\beta} = 0 \\
[A_{b,4}]: &\qquad 1 - \lambda_{1,4} + \lambda_{2,4} - \lambda_{3,4} = 0 \\
[D_{s,4}]: &\qquad D_{s,4} = \frac{1 + \lambda_{4,4}}{2\psi_s\left(1-\lambda_{3,4}\right)} \\
[D_{b,4}]: &\qquad D_{b,4} = \frac{\lambda_{1,4} + \lambda_{5,4}}{2\psi_b\left(1-\lambda_{3,4}\right)} \\
\left[\bar{T}_4\right]: &\qquad \lambda_{8,4} = \beta\left(1-p\right)\left(1-\lambda_{1,4}\right) \\
\left[\underline{T_4}\right]: &\qquad \lambda_{6,4} = -\beta p\left(1-\lambda_{1,4}\right)
\end{aligned}
$$

Simplifying the FOCs and adding the constraints yields the following system:

$$
\begin{aligned}
[L_4]: &\quad L_4 = \frac{\beta\lambda_{1,4}\tilde{z} + \bar{z}\beta\left(1-p\right)\left(1-\lambda_{1,4}\right)}{(1-\lambda_{3,4})w} \\
[B_{s,4}]: &\quad 1 + \lambda_{3,4} + \lambda_{4,4} - p\left(1-\lambda_{1,4}\right) = 0 \\
[B_{b,4}]: &\quad 1 + \lambda_{2,4} + \lambda_{5,4} - p\left(1-\lambda_{1,4}\right) = 0 \\
[A_{b,4}]: &\quad \lambda_{3,4} - \lambda_{2,4} = 1 - \lambda_{1,4} = \lambda_{5,4} - \lambda_{4,4} \\
[D_{s,4}]: &\quad D_{s,4} = \frac{1+\lambda_{4,4}}{2\psi_s\left(1-\lambda_{3,4}\right)} = \frac{p\left(1-\lambda_{1,4}\right)-\lambda_{3,4}}{2\psi_s\left(1-\lambda_{3,4}\right)} \\
[D_{b,4}]: &\quad D_{b,4} = \frac{\lambda_{1,4}+\lambda_{5,4}}{2\psi_b\left(1-\lambda_{3,4}\right)} = \frac{1+\lambda_{4,4}}{2\psi_b\left(1-\lambda_{3,4}\right)} = \frac{p\left(1-\lambda_{1,4}\right)-\lambda_{3,4}}{2\psi_b\left(1-\lambda_{3,4}\right)} \\
\left[\bar{T}_4\right]: &\quad \lambda_{8,4} = \beta\left(1-p\right)\left(1-\lambda_{1,4}\right) \\
\left[\underline{T_4}\right]: &\quad \lambda_{6,4} = -\beta p\left(1-\lambda_{1,4}\right) \\
&\quad \tilde{z}\ln L_4 - D_{b,4}(1+r^*) - \tilde{T}_4 = \frac{\Gamma_b\left(\psi_b\right)}{\beta} \\
&\quad D_{b,4}\left(1+\frac{\psi_b}{\psi_s}\right) - \psi_b\left(D_{b,4}\right)^2 - \frac{1}{\psi_s}\left(\psi_b D_{b,4}\right)^2 - wL_4 - \xi = 0 \\
&\quad \underline{T_4} = D_{b,4}(1+r^*)\frac{\psi_b}{\psi_s} \\
&\quad \bar{z}\ln L_4 - \bar{T}_4 = D_{b,4}(1+r^*)
\end{aligned}
$$

Note that:

$$
\begin{aligned}
\tilde{T}_4 &= pD_{b,4}(1+r^*)\frac{\psi_b}{\psi_s} + \left(1-p\right)\bar{z}\ln L_4 - \left(1-p\right)D_{b,4}(1+r^*) \\
\tilde{T}_4 &= D_{b,4}(1+r^*)\left(p\frac{\psi_s+\psi_b}{\psi_s} - 1\right) + \left(1-p\right)\bar{z}\ln L_4
\end{aligned}
$$

Then using the first constraint we get:

$$
\begin{aligned}
p\underline{z}\ln L_4 - D_{b,4}(1+r^*)\left(p\frac{\psi_s+\psi_b}{\psi_s}\right) &= \frac{\Gamma_b\left(\psi_b\right)}{\beta} \\
\Rightarrow \quad \ln L_4 &= \frac{D_{b,4}(1+r^*)p\left(1+\frac{\psi_b}{\psi_s}\right)+\frac{\Gamma_b(\psi_b)}{\beta}}{p\underline{z}} \\
\Rightarrow \quad wL_4 &= we^{\frac{D_{b,4}(1+r^*)p\left(1+\frac{\psi_b}{\psi_s}\right)+\frac{\Gamma_b(\psi_b)}{\beta}}{p\underline{z}}}
\end{aligned}
$$

The above derivations arrive at the solution to this problem as described in the main text. $D_{s,4}$ is characterized by the implicit equation in expression (27). Expression (27) resembles closely the equilibrium expression (53) for Case 3 above. Once again, the quadratic equation in expression (27) has either no solution or two solutions (with a knife edge case of a unique solution). The max of the LHS is still $D_s = \frac{1}{2\psi_s}$. However the RHS of expression (27) is clearly larger than the RHS in expression (53) as long as the buyer's outside option is strictly positive. Evaluating the LHS and the RHS at the maximum value obtains:

$$
\begin{aligned}
LHS &= \frac{1}{4}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right) \\
RHS &= we^{\frac{\frac{1+r^*}{2}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)}{\underline{z}}+\frac{\Gamma_b(\psi_b)}{\beta p\underline{z}}}+\xi
\end{aligned}
$$

To ensure the existence of a pair of solutions, the following parameter restriction is necessary:

$$
\frac{1}{4}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right) \geq we^{\frac{1+r^*}{2\underline{z}}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)+\frac{\Gamma_b(\psi_b)}{\beta p\underline{z}}}+\xi \tag{62}
$$

Given the two roots, the optimal level of $D_{s,4}$ is the one that is associated with a higher value function for the seller. To see when that occurs, substitute constraints (1), (2) and (3) into the seller's objective function to obtain

$$V_{s,4} = -(D_{s,4} + D_{b,4})(1 + r^*) + \tilde{z} \ln L_4 - \Gamma(\psi_b)/\beta$$

Substituting out $L_4$ using expression (26) in the above and using the proportionality result between the debt levels shows that the seller's value function is maximized whenever $D_{s,4}$ is maximized because $\bar{z} > \underline{z}$. Hence, the higher value of debt is once again the optimal solution to this case. Furthermore, the shapes of the RHS and the LHS allow us to derive some useful comparative statics. In particular, the LHS is a parabola with an inverted U shape as a function of $D_{s,4}$ while the RHS is an increasing exponential. Since the solution is the larger of the two roots, any parameter that shifts the RHS up results in a lower optimal debt level. In particular, suppose a parameter raises the buyer's outside option, $\Gamma_b(\psi_b)$. This necessarily lowers the optimal debt level.

Given the discussion above, the maximized value of the seller is[12]:

$$V_{s,4} = D_{b,4}(1 + r^*)\frac{\psi_s + \psi_b}{\psi_s}(1 - p)\left(\frac{\bar{z}}{\underline{z}} - 1\right) + \frac{(1 - p)\bar{z}}{p\underline{z}}\Gamma(\psi_b)/\beta$$

Comparing cases 3 and 4, clearly case 4 yields a lower value of debt because the RHS in this case is strictly higher. Since labor is increasing in debt, production is also lower in this case.

[12]The expression suggests that the seller's value function is increasing in the outside option of the buyer, $\Gamma_b(\psi_b)$, which is counter-intuitive. But that is not the case. There are two countervailing forces of $\Gamma_b(\psi_b)$ on $V_{s,4}$. The second part of the expression is clearly increasing in $\Gamma_b(\psi_b)$. However, the first part of the expression is increasing in $D_{b,4}$, which is a decreasing function of $\Gamma_b(\psi_b)$ since $D_{b,4}$ is proportional to $D_{s,4}$. Taking the derivative of $V_{s,4}$ with respect to $\Gamma_b(\psi_b)$ and applying the Implicit Function Theorem to expression (27) yields the following condition to ensure that $\frac{\partial D_{s,4}}{\partial \Gamma_b(\psi_b)} < 0$: $we^{\frac{(1+r^*)\left(1+\frac{\psi_s}{\psi_b}\right)D_{s,4}}{\underline{z}} + \frac{\Gamma_b(\psi_b)}{\beta p \underline{z}}} < \beta\bar{z}(1 - 2\psi_s D_{s,4})$.

## B.6 Existence of Different Cases

### B.6.1 Cases Without Insurance

Whether the equilibrium is as in Case 1 or as in Case 2, depends on whether the seller's value function dominates in the first or the second case. The difference in optimized value functions, $V_{s,1} - V_{s,2}$, is positive only if

$$\tilde{z}\ln\left(\frac{\tilde{z}\beta}{w}\right) - \underline{z}\ln\left(\frac{\underline{z}\beta}{w}\right) + \frac{\psi_b + \psi_s}{2\beta\psi_s\psi_b}\left[\frac{\tilde{\lambda}_1}{\tilde{\lambda}_1 - 1} - \frac{\tilde{\lambda}_2}{\tilde{\lambda}_2 - 1}\right] - (\tilde{z}\ln(1 - \tilde{\lambda}_1) - \underline{z}\ln(1 - \tilde{\lambda}_2)) > \Gamma_b(\psi_b)/\beta \tag{63}$$

Recall that case 1 is feasible when restrictions (58)-(60) hold. These restrictions can coexist with restriction (63). First, note that:

$$\tilde{z}\ln\left(\frac{\tilde{z}\beta}{w}\right) - \underline{z}\ln\left(\frac{\underline{z}\beta}{w}\right) + \frac{\psi_b + \psi_s}{2\beta\psi_s\psi_b}\left[\frac{\tilde{\lambda}_1}{\tilde{\lambda}_1 - 1} - \frac{\tilde{\lambda}_2}{\tilde{\lambda}_2 - 1}\right] - (\tilde{z}\ln(1 - \tilde{\lambda}_1) - \underline{z}\ln(1 - \tilde{\lambda}_2)) >$$
$$(\tilde{z} - \underline{z})\ln\left(\frac{\tilde{z}\beta}{w(1 - \tilde{\lambda}_1)}\right)$$

Also, restriction (58) constitutes the seller's maximized value under case 1, while restriction (63) is the difference in maximized values between cases 1 and 2. By construction, it must be that restriction (63) is more binding. But when

$$\Gamma_b(\psi_b)/\beta < (\tilde{z} - \underline{z})\ln\left(\frac{\tilde{z}\beta}{w(1 - \tilde{\lambda}_1)}\right) \tag{64}$$

case 1 is ruled out because it is not feasible. So case 2 dominates case 1 when (64) holds. But case 1 dominates case 2 when

$$\tilde{z}\ln\left(\frac{\tilde{z}\beta}{w}\right)-\underline{z}\ln\left(\frac{\underline{z}\beta}{w}\right)+\frac{\psi_b+\psi_s}{2\beta\psi_s\psi_b}\left[\frac{\lambda}{\lambda-1}-\frac{\lambda_2}{\lambda_2-1}\right]-\left(\tilde{z}\ln(1-\lambda)-\underline{z}\ln(1-\lambda_2)\right)>$$
$$\Gamma_b(\psi_b)/\beta\geq(\tilde{z}-\underline{z})\ln\left(\frac{\tilde{z}\beta}{w(1-\lambda)}\right)$$

and (60) hold, excluding the range in (64).

There is also a range of parameters in which case 2 is not feasible but case 1 is, so case 1 dominates case 2 in that range. To find that range, note that, in order for there to exist an equilibrium, the debt levels and hours worked must be non-negative. The joint restriction for these variables implies that the Lagrange multiplier $\tilde{\lambda}_1$ (or $\tilde{\lambda}_2$) must be non-positive. Since $\tilde{\lambda}_2>\tilde{\lambda}_1$, it must be that $\tilde{\lambda}_2$ attains the zero upper bound first. At that point, case 2 becomes infeasible. This occurs when $\tilde{\lambda}_2\geq 0$. Using the expression for $\tilde{\lambda}_2$, the range must be

$$\frac{\psi_s+\psi_b}{\psi_s\psi_b}\geq 2\left(\underline{z}\beta\right)+\sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2+4\left(\underline{z}\beta\right)^2}\qquad(65)$$

In order for case 1 to remain feasible, it must be that $\tilde{\lambda}_1\leq 0$. Using the expression for $\tilde{\lambda}_1$, the range must be

$$\frac{\psi_s+\psi_b}{\psi_s\psi_b}\leq 2\left(\tilde{z}\beta\right)+\sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2+4\left(\tilde{z}\beta\right)^2}\qquad(66)$$

Combining expressions (65) and (66) yields

$$2\left(\tilde{z}\beta\right)+\sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2+4\left(\tilde{z}\beta\right)^2}\geq\frac{\psi_s+\psi_b}{\psi_s\psi_b}\geq 2\left(\underline{z}\beta\right)+\sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2+4\left(\underline{z}\beta\right)^2}\qquad(67)$$

For as long as parameters satisfy the restriction in expression (67), case 2 is not feasible, case 1 is feasible (subject to the restriction in expression (64) which can co-exist), and case 1 dominates case 2.

### B.6.2 Cases With Insurance

In this section, we show that equilibria with insurance can arise. Recall that, cases 3 and 4 correspond to situations where agents use trade credit for insurance purposes. First, we show that cases 3 and 4 can dominate case 2 under certain conditions. Second, we show that cases 3 and 4 can dominate case 1 under certain conditions. Finally, we derive conditions that guarantee that case 3 dominates case 4 and vise verse.

**Cases 3 and 4 vs. Case 2.** We want to show that there exist an equilibrium in which the transfers are state contingent. To do that, we show that, starting with an equilibrium as in case 2 where the transfers are equalized, there exists a deviation that yields higher utility for the seller and is still feasible. Feasibility will be determined by a set of parameters.

In case 2, we know that constraint (7) binds, but constraints (1), (6) and (8) do not. The seller would like to extract all surplus from the buyer if possible, i.e. they would like to make constraint (1) bind. They can do so for given $D_{b,2}$ and $L_2$ by increasing the average transfer. However, in an equilibrium where the transfers are state-contingent, increasing the mean transfer (by either increasing the transfer in the good state or lowering it in the bad state) implies that they would have to pay the fixed cost $\xi$. This cost is financed with debt, which mean that the seller would have to increase the amount that they borrow. Since their surplus is increasing in the mean transfer and falling in the amount of debt, the deviation will only be profitable under some parameter values.

Suppose they want to expropriate all of the buyer's surplus. Then the mean transfer that they receive would have to increase by:

$$\epsilon = (\tilde{z} - \underline{z}) \ln L_2 - \Gamma(\psi_b)/\beta, \tag{68}$$

which corresponds to the value such that constraint (1) binds for given scale of production $L_2$. The seller is better off if this gain surpasses the loss due to higher debt:

$$\epsilon > x_d \tag{69}$$

where $x_d$ is the extra debt, in addition to $D_{s,2}$, needed in order to cover all costs, keeping $A_{b,2}$ and $L_2$ fixed, and satisfies

$$D_{s,2} + x_d = \xi + \psi_s[(D_{s,2} + x_d)^2 - d_s^2] \tag{70}$$

This quadratic equation has two roots. We take the smaller of the two roots because we want to minimize the amount of debt raised. The solution is:

$$x_d = \frac{1 - 2\psi_s D_{s,2} - \sqrt{(2\psi_s D_{s,2} - 1)^2 - 4\psi_s(\xi - D_{s,2})}}{2\psi_s} \tag{71}$$

Notice that $x_d$ is increasing in $\xi$. We can find a sufficient condition to ensure that $x_d > 0$. First, note that the upper bound for $D_{s,2} = \frac{1}{2\psi_s}$ which occurs as $\tilde{\lambda}_2 \to -\infty$. At the upper bound, $x_d < 0$. The lower bound for $D_{s,2} = 0$ which occurs as $\tilde{\lambda}_2 \to 0$. At this bound,

$$x_d = \frac{1 - \sqrt{1 - 4\psi_s \xi}}{2\psi_s}, \tag{72}$$

which is positive if and only if

$$0 < \xi < \frac{1}{4\psi_s}. \tag{73}$$

Hence, there exists small enough $\xi > 0$ such that $x_d > 0$. Since $x_d$ is decreasing in $\xi$ and $\epsilon$ is independent of $\xi$, there exists a small enough $\xi > 0$ such that $\epsilon > x_d$. Hence, cases 3 and 4 exist against case 2.

**Cases 3 and 4 vs. Case 1** A sufficient condition to ensure that case 3 dominates case 1 is that case 1, which is the unconstrained equilibrium, is not feasible. Case 1 is not feasible whenever constraint (7) is violated. Substituting the expression for $T_1$ and $L_1$ into this constraint yields the following parameter restriction which ensures that case 3 dominates case 1:

$$\frac{\Gamma_b(\psi_b)}{\beta} < (\tilde{z} - \underline{z}) \ln \left( \frac{\tilde{z}\beta(\psi_s + \psi_b)}{w\psi_s\psi_b \left[ 2\left(\tilde{z}\beta\right) + \sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\tilde{z}\beta\right)^2} \right]} \right) \tag{74}$$

The same restriction ensures that case 4 is preferred to case 1 because constraint (7) is slack in case 4.

**Case 4 vs. Case 3** Comparing the value functions in the two cases, $V_{s,4} > V_{s,3}$ if and only if:

$$\frac{\Gamma_b(\psi_b)}{\beta p \underline{z}} > \frac{\psi_b + \psi_s}{\psi_b} \frac{(1-p)}{\beta \tilde{z}} \left( \frac{\bar{z}}{\underline{z}} - 1 \right) (D_{s,3} - D_{s,4}) > 0 \tag{75}$$

The difference $D_{b,3} - D_{b,4}$ is increasing in $\frac{\Gamma_b(\psi_b)}{\beta p \underline{z}}$. However, there exist parameter restrictions that ensure that this increase is less than linear such that the inequality in (75) holds. Consider the limiting case of $\xi = \epsilon > 0$, where $\epsilon$ is a small positive number. Taking logs of expression (27) yields the following log approximation of the implicit function that defines $D_{s,4}$:

$$\log \left( (1 - \psi_s D_{s,4}) \left( 1 + \frac{\psi_s}{\psi_b} \right) D_{s,4} \right) \approx \log w + \frac{(1 + r^*) \left( 1 + \frac{\psi_s}{\psi_b} \right)}{\underline{z}} D_{s,4} + \frac{\Gamma_b(\psi_b)}{\beta p \underline{z}} \tag{76}$$

The LHS of expression (76) is a parabola that has an inverted U shape, while the RHS is linear in $D_{s,4}$. Since $\frac{\Gamma_b(\psi_b)}{\beta p \underline{z}}$ enters the intercept of the RHS, an increase shifts the RHS up by that amount. However, since the LHS has curvature, the decline in $D_{s,4}$, $\Delta D_{s,4}$, is strictly less than $\frac{\Gamma_b(\psi_b)}{\beta p \underline{z}}$. From expressions (53) and (27), $\Delta D_{s,4} = D_{s,3} - D_{s,4}$. All that remains is to ensure that the proportionality factor that scales this difference in expression (76) is not very large. A sufficient condition is that the following parameter restriction holds:

$$\frac{\psi_b + \psi_s}{\psi_b} \frac{(1-p)}{\beta \tilde{z}} \left( \frac{\bar{z}}{\underline{z}} - 1 \right) \leq 1 \tag{77}$$

Hence, case 4 dominates whenever restriction (77) holds for small $\xi > 0$.

Comparing expressions (61) and (62), clearly the conditions for existence of case 3 are less strict than those for case 4. Hence, case 3 dominates at the minimum whenever case 4 is not feasible, which occurs when

$$we^{\frac{1+r^*}{2\underline{z}}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)+\frac{\Gamma_b(\psi_b)}{\beta p \underline{z}}} + \xi > \frac{1}{4}\left(\frac{1}{\psi_s} + \frac{1}{\psi_b}\right) \geq we^{\frac{1+r^*}{2\underline{z}}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)} + \xi \tag{78}$$